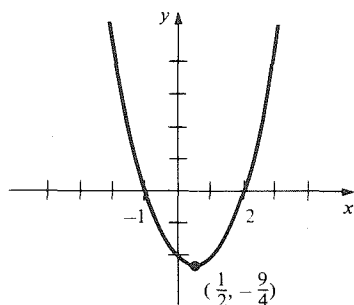


# Answers to Orientation Quizzes

## Answers to Quiz A on p. 13

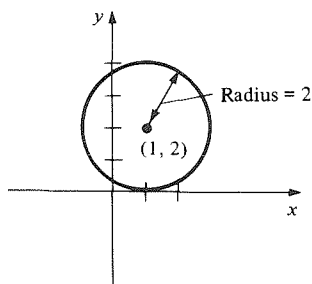
1.  $-\frac{4}{3}$     2.  $x > -\frac{2}{3}$     3.  $x < -\frac{1}{3}$ ,  $x > 1$     4.  $x = -2 \pm \frac{\sqrt{38}}{2}$

5.



6.  $g(2) = \frac{4}{3}$ . The domain of  $g$  is all  $x$  such that  $x \neq 0, \frac{1}{2}$ .    7.  $x > -10$     8. At the points  $(1, 1)$  and  $(2, 4)$

9.



10. 2

## Answers to Quiz B on p. 14

1.  $\frac{11}{4} = 2\frac{3}{4}$     2.  $x(x+3)$     3. 10    4. 4 kilograms    5.  $(2x-1)/2x$  (or  $1 - 1/2x$ )    6.  $x^7$   
7.  $-6, -4, 0, \frac{1}{2}, 8$     8.  $-\frac{3}{7}$     9.  $\frac{1}{3}$     10.  $x+4$

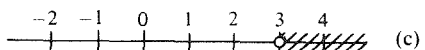
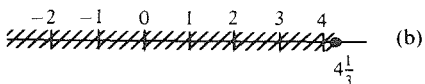
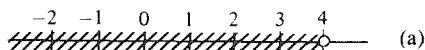
## Answers to Quiz C on p. 14

1.  $(-2, -3)$     2.  $\frac{8}{7}$     3.  $150^\circ$     4.  $\frac{4}{5}$     5.  $\sqrt{5}$     6.  $8\pi$  centimeters    7. 14 meters  
8.  $18\pi$  cubic centimeters    9.  $\frac{1}{2}$     10. 2

# Chapter R Answers

## R.1 Basic Algebra: Real Numbers and Inequalities

1. Rational
5.  $ab - 5b - 3c$
9.  $a^3 - 3a^2b + 3ab^2 - b^3$
11.  $b^4 + 4b^3c + 6b^2c^2 + 4bc^3 + c^4$
13.  $(x + 3)(x + 2)$
17.  $3(x - 4)(x + 2)$
21. 2
25. Multiply out  $(x - 1)(x^2 + x + 1)$
27.  $x(x + 2)(x - 1)$
29. -4, -1
33.  $(5 \pm \sqrt{26.2})/2$
37.  $(1 \pm \sqrt{33})/4$
41.  $\sqrt{7}/2$
43.  $-\sqrt{3}, -5/3, -\sqrt{2}, -7/5, 0, 9/5, 23/8, 3, 22/7$
45.  $a > c$
47.  $b^2 < c$
49.  $-2 > b$
51. (a)  $x < 4$   
(b)  $x \leq 13/3$   
(c)  $x > 3$



53. (a)

$$ax^2 + bx + c = a \left[ \left( x + \frac{b}{2a} \right)^2 + \left( \frac{c}{a} - \frac{b^2}{4a^2} \right) \right] = 0$$

$$\text{has solution } x + b/2a = \pm \sqrt{\left( \frac{c}{a} - \frac{b^2}{4a^2} \right)}. \text{ Now}$$

simplify.

(b)  $b^2 = 4ac$  exactly when the discriminant vanishes.

## R.2 Intervals and Absolute Values

1. (a) True (b) False (c) True  
(d) False (e) True
3. If  $a < b$ , then  $a = 3a - 2a < b$ , so  $3a < 2a + b$  and  $a < (2a + b)/3$ .
5.  $[3, \infty)$
7.  $(-\infty, 2)$
9.  $(-\infty, -3)$  or  $(1, \infty)$
11.  $(-\infty, 0]$  or  $[1, \infty)$
13. 2
15. 8
17. 15
19. 15

15. 8

19. 15

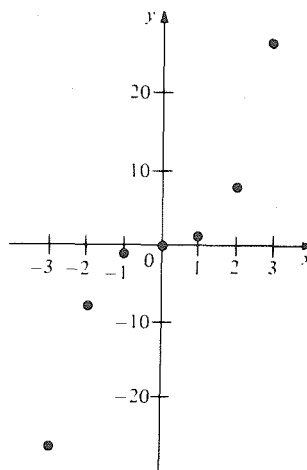
21.  $\pm 8$
23.  $|x + 5/2| > 5/2$
25.  $|x - 1/2| > 3/2$
27.  $(3, 4]$
29.  $(-5, 5)$
31.  $(-1, 1/3)$
33.  $|x| < 3$
35.  $|x| < 6$
37.  $|x - 2| \leq 10$
39. Let  $x = 1, y = 1$ , and  $z = -1$ .
41. Take the cases  $x \geq 0$  and  $x < 0$  separately.
43. No

## R.3 Laws of Exponents

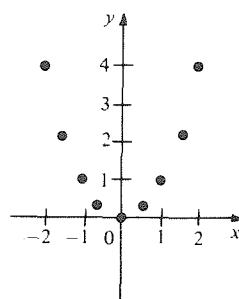
1. 1
3.  $64(4 \cdot 3^{10} + 1)$
5.  $1/(2^9 \cdot 3^{12})$
7.  $4^4$
9. 3
11. 3
13.  $1/8$
15. 36
17.  $1 + x$
19.  $x + 2x^3 + 3x^6$
21. Use  $\sqrt[a]{x} = x^{1/a}$
23.  $(ab)^{3/2}$
25. 4, 8, 32
27.  $(x^{1/2} - 2y^{1/2})(x^{1/2} + y^{1/2})$
29.  $(x^{1/2} - 4)(x^{1/2} + 2)$
31. Both choices are consistent.
33. Proceed as in the proof that  $b^{p+q} = b^p b^q$ .

## R.4 Straight Lines

1.

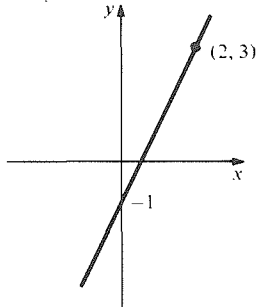


3.

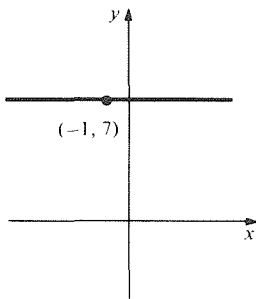


5. 2  
 9. 43718  
 13.  $2\sqrt{x^2 + 25}$   
 17. 3  
 7.  $\sqrt{314}$   
 11.  $\sqrt{2}$   
 15.  $2a\sqrt{2}$   
 19.  $-7/2$

21.  $y = 2x - 1$



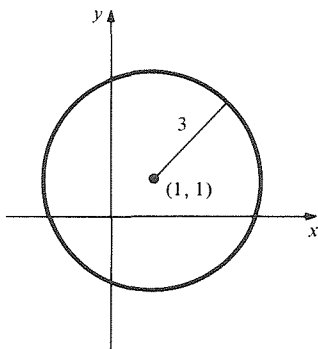
23.  $y = 7$



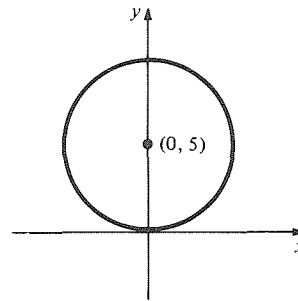
25.  $2y = x + 9$   
 27.  $y = 4$   
 29.  $m = -1/2$ , intercept is  $-2$   
 31.  $m = 0$ , intercept is  $17/4$   
 33.  $m = -11/7$ , intercept is  $13/7$   
 35.  $m = 0$ , intercept is  $17$   
 37. (a)  $m = -4/5$   
 (b)  $4y = 5x - 1$   
 39.  $y = 5x + 14$   
 41.  $y = -x + 6$   
 43.  $(1, -3)$ ,  $(1 + 2\sqrt{6}, 3)$  or  $(1 - 2\sqrt{6}, 3)$

## R.5 Circles and Parabolas

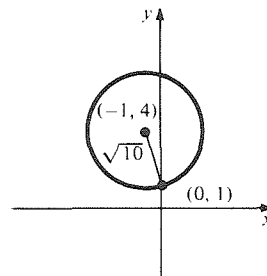
1.  $(x - 1)^2 + (y - 1)^2 = 9$



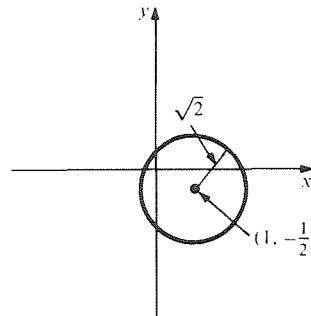
3.  $x^2 + (y - 5)^2 = 25$



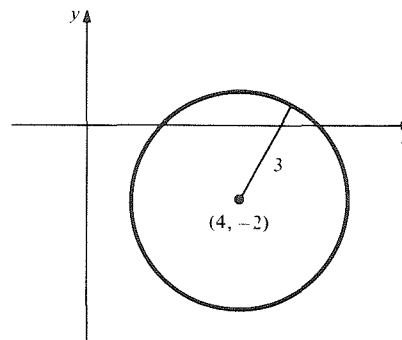
5.  $(x + 1)^2 + (y - 4)^2 = 10$



7.  $r = \sqrt{2}$ , center is  $(1, -1/2)$



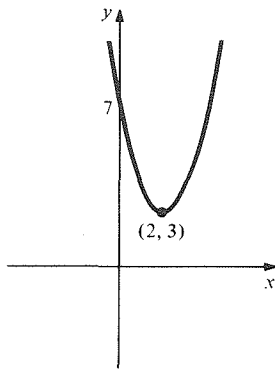
9.  $r = 3$ , center is  $(4, -2)$



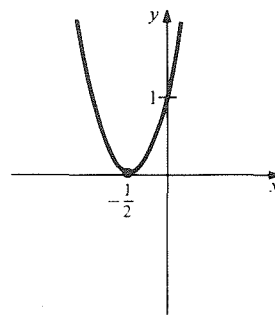
11.  $y = -x^2 + 2x + 1$   
 13.  $y = -x^2/5 + 2x$

A.4 Chapter R Answers

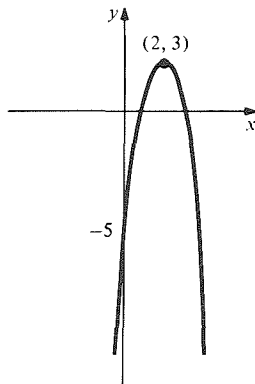
15.



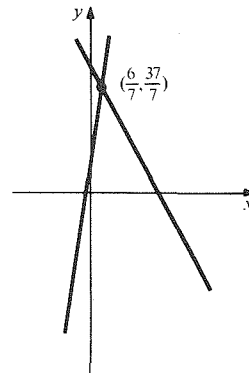
23.



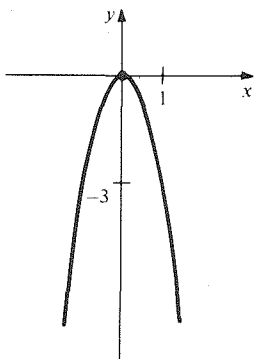
17.



25.  $(\frac{6}{7}, \frac{37}{7})$

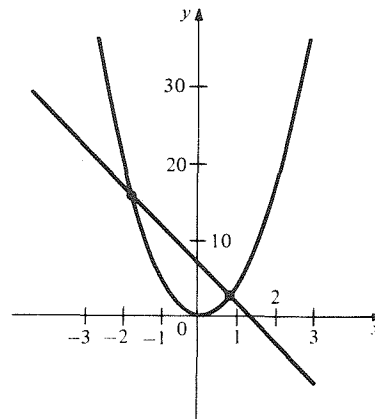
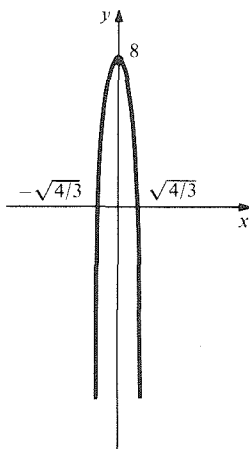


19.

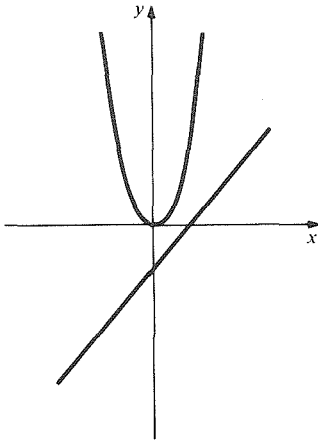


$$27. \left( \frac{-3 + 2\sqrt{11}}{5}, \frac{53 - 12\sqrt{11}}{5} \right), \left( \frac{-3 - 2\sqrt{11}}{5}, \frac{53 + 12\sqrt{11}}{5} \right)$$

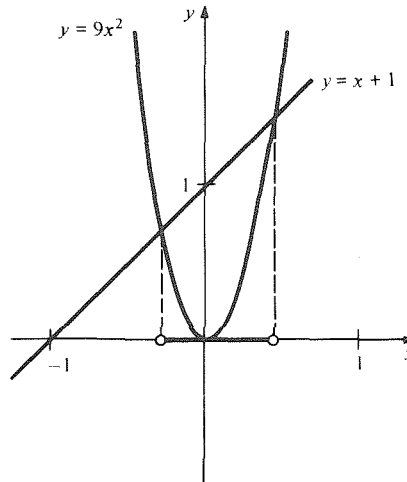
21.



29. No intersection.



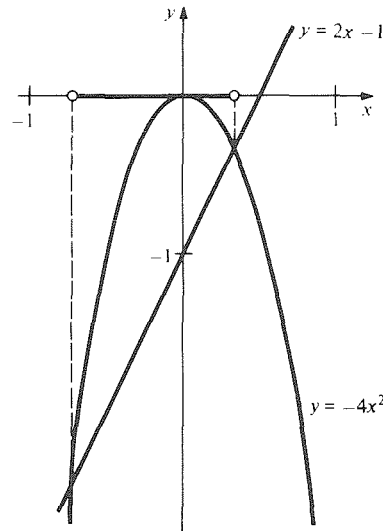
37.  $x \in ((1 - \sqrt{37})/18, (1 + \sqrt{37})/18)$



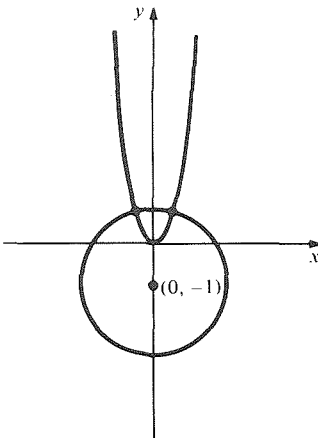
31. 0, 1, 2, or infinitely many intersections.



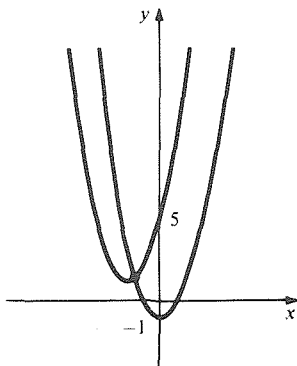
39.  $x \in ((-1 - \sqrt{5})/4, (-1 + \sqrt{5})/4)$



33.  $\left(\sqrt{(-9 + \sqrt{273})/32}, (-9 + \sqrt{273})/8\right)$   
and  $\left(-\sqrt{(-9 + \sqrt{273})/32}, (-9 + \sqrt{273})/8\right)$

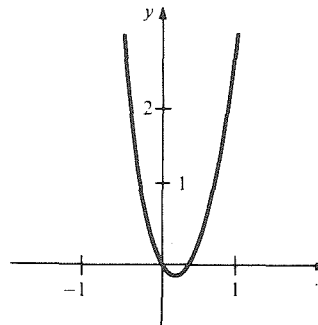


35.  $(-3/2, 5/4)$



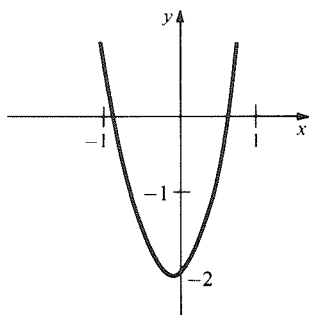
## R.6 Functions and Graphs

1. 7; 3
3. -2; 0
5. 4; 0
7. Domain is  $x \neq 1$ ;  $f(10) = 100/9$ .
9. Domain is  $-1 < x < 1$ ;  $f(10)$  does not exist.
11. Domain is  $x \neq -2, 3$ ;  $f(10) = 13/21$ .
- 13.

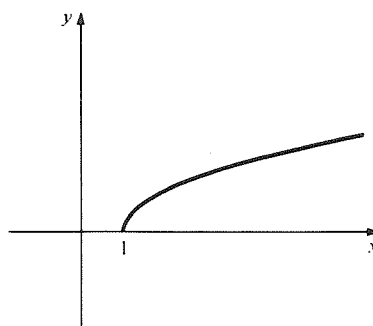


**A.6 Chapter R Answers**

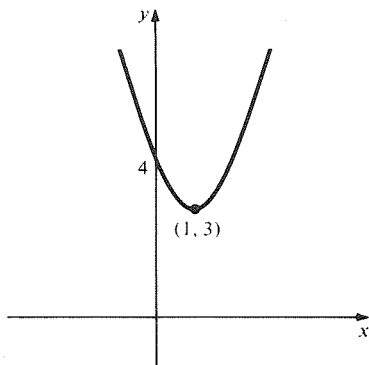
15.



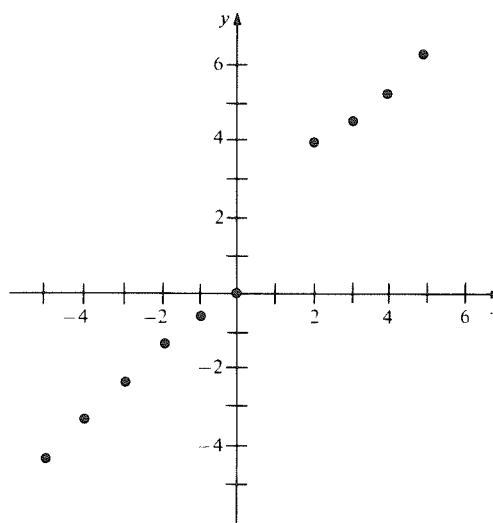
23.



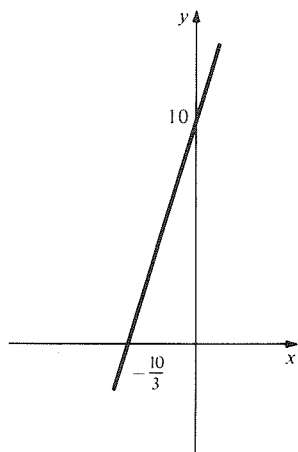
17.



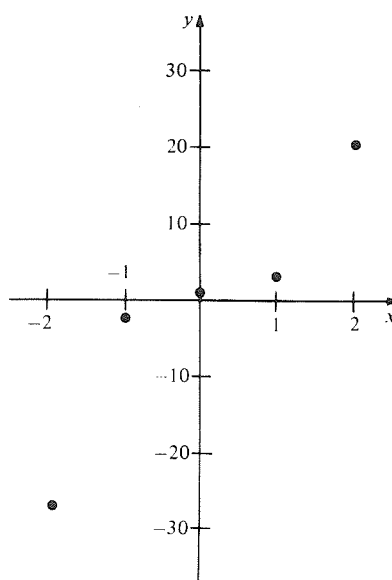
25.



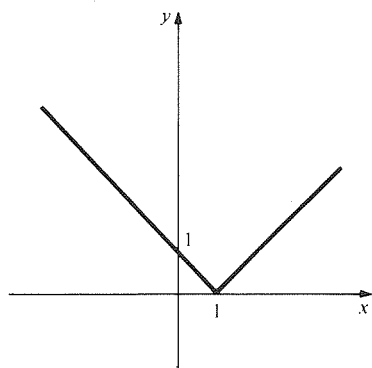
19.



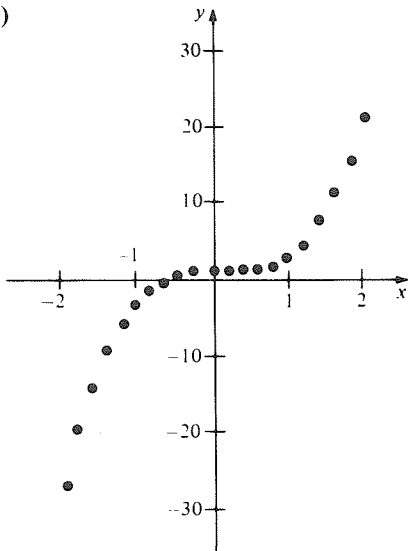
27. (a)



21.



27. (b)



29. a, c

31. Yes; domain is  $x \neq 0$ .33. Yes; domain is  $x \leq -1$  and  $x \geq 1$ .

35. Yes; domain is all real numbers.

## Review Exercises for Chapter R

1.  $-2/3$

5.  $\frac{1}{2}$

9.  $x > -1/4$

13.  $x > 3/2$

17.  $(-1, 1)$

21.  $(-\infty, 1 - \sqrt{2}]$  and  $[1 + \sqrt{2}, \infty)$

25.  $(-2, 3)$

29.  $(-\infty, 13/2]$  and  $(22/3, \infty)$

31.  $(-\infty, -5]$ ,  $(1, 5/2)$  and  $[3, \infty)$

33. 13

37. 1

41.  $\sqrt[4]{xy}$

45.  $\sqrt{10}$

49.  $13y - 8x + 17 = 0$

53.  $4y + 12x - 61 = 0$

57.  $8y - 5x - 3 = 0$

61.  $x^2 - 24x + y^2 - 10y + 105 = 0$

63.  $x^2 - 14x + y^2 + 2y + 41 = 0$

3.  $(1 \pm \sqrt{5})/2$

7. 3

11.  $-\sqrt{5} < x < \sqrt{5}$

15.  $-\sqrt{7/3} < x < \sqrt{7/3}$

19.  $(0, 2)$

23.  $(5, 6)$

27.  $[5, 6)$

35. 8

39. 4

43.  $(\sqrt{x-1} - \sqrt{x-8})/7$

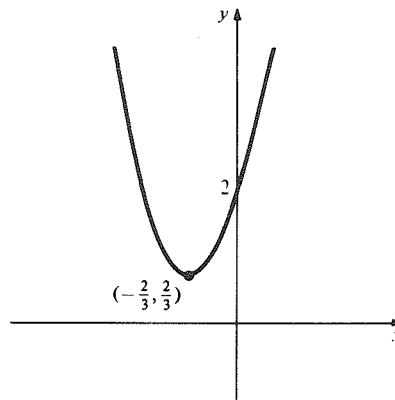
47.  $5\sqrt{2}$

51.  $x + y = 1$

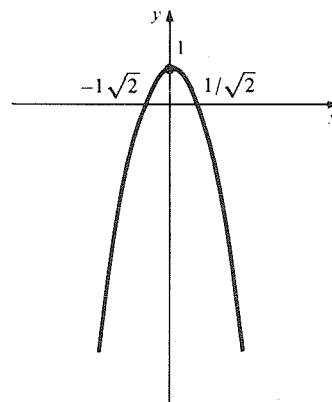
55.  $2y - 7x - 34 = 0$

59.  $y - 4 = 0$

65.



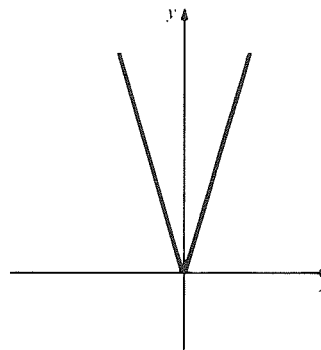
67.



69.  $(-\sqrt{2}, -\sqrt{2})$ ,  $(\sqrt{2}, \sqrt{2})$

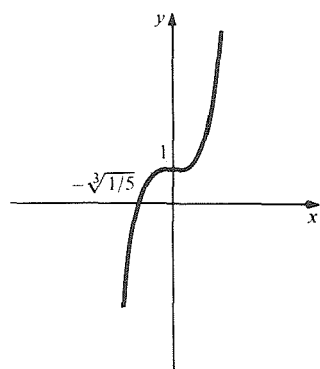
71. No intersection.

73.

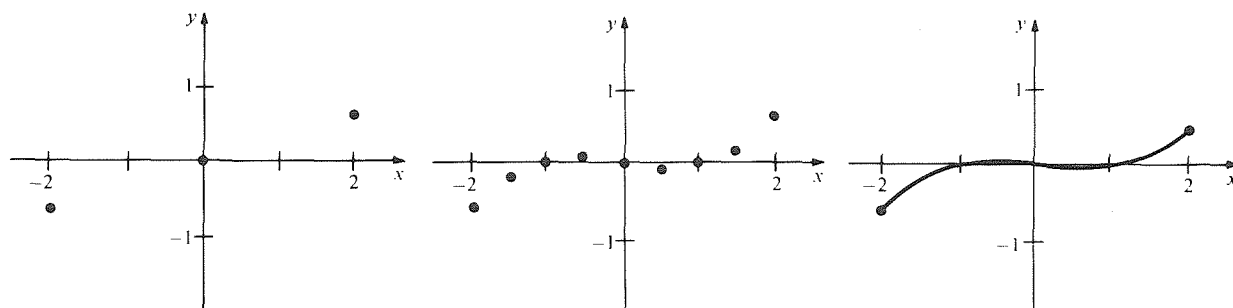


# A.8 Chapter R Answers

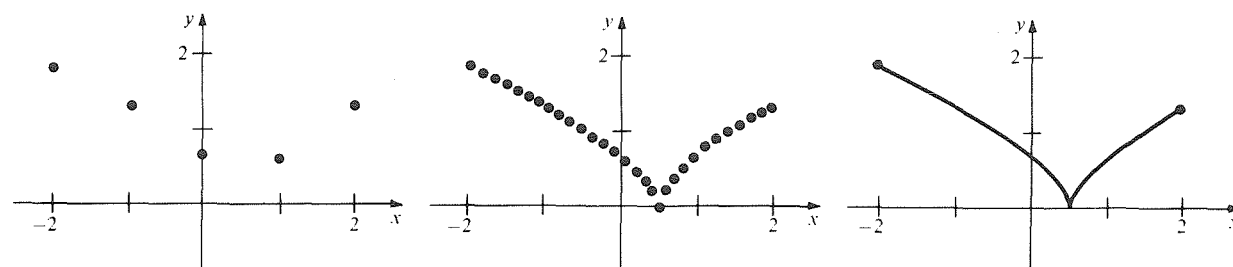
75.



77.



79.



81.  $a, c$

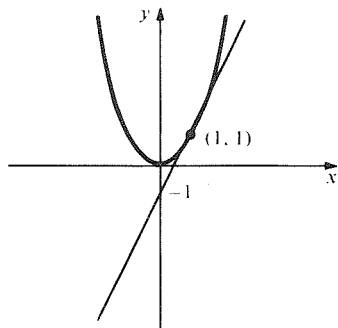
83. (a)  $\frac{1}{1 + 1/(k+l)}$  has the smaller denominator

(b) use  $\frac{1}{1 + 1/l} = \frac{l}{1+l}$ .

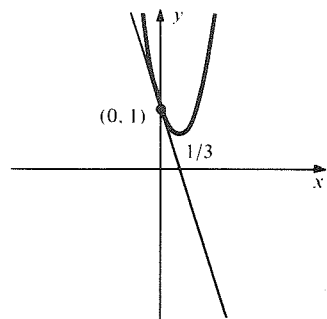
# Chapter 1 Answers

## 1.1 Introduction to the Derivative

1. (a)  $\Delta y = 3.75$ ;  $\Delta y/\Delta x = 7.5$   
 (b)  $\Delta y = 0.0701$ ;  $\Delta y/\Delta x = 7.01$   
 (c)  $\Delta y = 1.11$ ;  $\Delta y/\Delta x = 11.1$   
 (d)  $\Delta y = 0.1101$ ;  $\Delta y/\Delta x = 11.01$
3. (a)  $\Delta y = 7.25$ ;  $\Delta y/\Delta x = 14.5$   
 (b)  $\Delta y = 0.1401$ ;  $\Delta y/\Delta x = 14.01$   
 (c)  $\Delta y = 1.81$ ;  $\Delta y/\Delta x = 18.1$   
 (d)  $\Delta y = 0.1801$ ;  $\Delta y/\Delta x = 18.01$
5. 7 meters/second
7. 13 meters/second
9. (a)  $2x_0 + 3$  (b)  $x_0 = 7/2$
11. (a)  $2x_0 + 10$  (b)  $x_0 = 0$
13. Slope is 2.

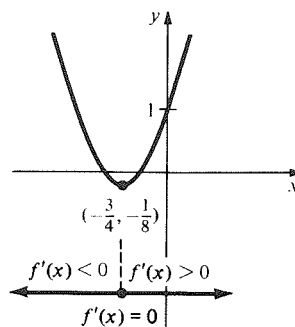


15. Slope is  $-3$ .



- |                |                         |
|----------------|-------------------------|
| 17. $g$        | 19. $2a$                |
| 21. 3          | 23. $-11$               |
| 25. $(8, -62)$ | 27. $(-2, 7)$           |
| 29. $2x + 3$   | 31. $2x$                |
| 33. $-8t + 3$  | 35. $-2s$               |
| 37. $t = 1$    | 39. 29.4 meters/second. |
| 41. 6          | 43. 0                   |
| 45. $-5$       | 47. $-1$                |
| 49. $8x + 3$   | 51. $-2x$               |
| 53. $-4x + 5$  |                         |

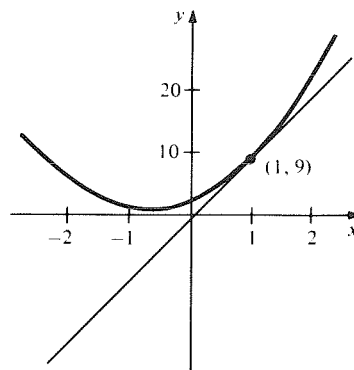
55.  $f'(x) = 0$  at  $x = -3/4$ ;  $f'(x) > 0$  for  $x > -3/4$ ;  
 $f'(x) < 0$  for  $x < -3/4$



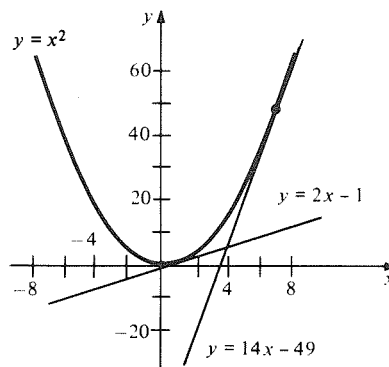
57.  $A'(x) = 2x$

59.  $(1, 0)$

61.  $y = 10x - 1$

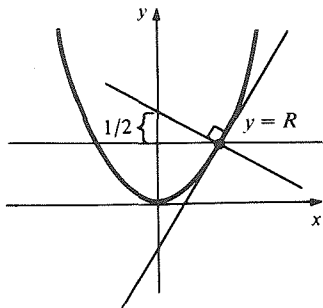


63.  $y = 14x - 49$  and  $y = 2x - 1$

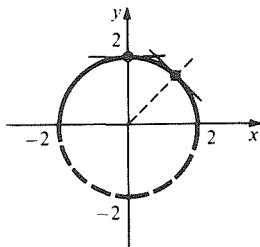


## A.10 Chapter 1 Answers

65. The equations of the lines are  $y = x_0^2$  and  $y = x_0^2 - (1/2x_0)(x - x_0)$



67.  $f'(0) = 0$ ;  $f'(\sqrt{2}) = -1$



69. (a) 24.5, 14.7, 4.9, -4.9, -14.7, -24.5 meters/sec.  
(b) 2.5 seconds  
(c) 2.5 seconds  
(d) 5 seconds

## 1.2 Limits

1. Limit is 2.
5. 20
9. 0
13.  $\sqrt{3}/6$
17.  $1/2$
21. 2
25.  $-\sqrt{3} - 1$
29. No limit exists.
33.  $2/3$
37.  $1 = f(0)$ ; 2,  $f(1)$  does not exist; no limit,  $f(2) = 2$ ; 1,  $f(3)$  does not exist; no limit,  $f(4) = 2$ .
39. 2, 4
43. No limit exists.
47.  $+\infty$
51. 5
55. 9
59.  $f(1) = 5$
61. (a)  $f(T) = g(T) = 0$   
(b) A thin sheet of ice of area  $A$  could remain just before complete melting. Hence,  $\lim_{t \rightarrow T} g(t) = 0 = g(T)$ , but  $\lim_{t \rightarrow T} f(t) = A \neq f(T)$ .  
(c)  $\lim_{t \rightarrow T} [f(t) \cdot g(t)] = 0$   
(d)  $\lim_{t \rightarrow T} f(t) \cdot \lim_{t \rightarrow T} g(t) = 0$  as well.

63. By the quotient rule,

$$\lim_{x \rightarrow x_0} \left[ \frac{1}{f(x)} \right] = 1 / \lim_{x \rightarrow x_0} f(x).$$

As with the argument that  $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$ , we can conclude that  $\lim_{x \rightarrow x_0} [1/f(x)] = 0$ .

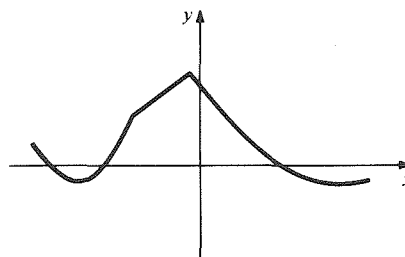
65. (a) Write  $f_1(x) + f_2(x) + f_3(x) + f_4(x) = [f_1(x) + f_2(x) + f_3(x)] + f_4(x)$ .  
(b) Write  $f_1(x) + f_2(x) + \cdots + f_{17}(x) = [f_1(x) + \cdots + f_{16}(x)] + f_{17}(x)$ .  
(c) Use the method of (b) with  $n$  in place of 16 and  $n+1$  in place of 17.  
(d) Choose  $m = 1$  and use the basic sum rule together with (c).

## 1.3 The Derivative as a Limit and the Leibniz Notation

1.  $2x + 1$
5.  $-3/x^2$
9.  $1/\sqrt{x}$
11.  $4x - 1/2\sqrt{x} - 1/x^2$
13. The difference quotient  $\frac{|\Delta x|}{\Delta x}$  has no limit at  $x_0 = 0$ . However,  $f(0) = 1$  and  $\lim_{x \rightarrow 0} (1 + |x|) = 1$  imply that  $f(x)$  is continuous at  $x_0 = 0$ .
15.  $2x - 1$
19.  $4/3$
23.  $-3$
27. 8
31.  $1/2$
3.  $15x^2$
7.  $2x - 3/x^2$
17.  $9x^2 + 1$
21. 15
25. 10
29.  $-2/x^3$
33. One answer is

$$f(x) = \begin{cases} 1 - x^2 & \text{for } -1 < x < 1 \\ 0 & \text{for } x \leq -1 \text{ or } x \geq 1 \end{cases}$$

Another is given in this figure:



35. (a) 1, 0  
(b)  $\frac{d}{dx} (\sin x) = \lim_{\Delta x \rightarrow 0} \frac{\sin(x + \Delta x) - \sin x}{\Delta x}$   
$$= \lim_{\Delta x \rightarrow 0} \frac{\sin x \cos \Delta x + \cos x \sin \Delta x - \sin x}{\Delta x}$$
$$= \lim_{\Delta x \rightarrow 0} \left[ \sin x \left( \frac{\cos \Delta x - 1}{\Delta x} \right) + \lim_{\Delta x \rightarrow 0} \left[ \cos x \left( \frac{\sin \Delta x}{\Delta x} \right) \right] \right]$$
$$= 0 + \cos x = \cos x.$$

## 1.4 Differentiating Polynomials

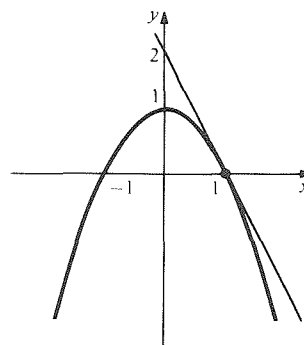
1.  $10x^9$
5.  $-20x^3$
9.  $3/2\sqrt{x}$
13.  $f'(x) + g'(x) = 6x + 1 = (f + g)'(x)$
15.  $f'(x) + g'(x) = 2x + 1 = (f + g)'(x)$
17.  $f'(x) - 2g'(x)$
21.  $5t^4 + 12t + 8$
25.  $13s^{12} + 96s^7 - 21s^6/8 + 4s^3 + 4s^2$
27.  $4x^3 - 9x^2 + 4x$
29.  $80h^9 + 9h^8 - 113h$
31.  $6x^5 + 12x^3 + 6x$
33.  $24t^7 - 306t^5 + 130t^4 + 48t^2 - 18t + 17$
35.  $-30r^5 + 20r^3 - 26r$
37.  $7t^6 - 5t^4 + 27t^2 - 9$
39.  $7u^6 + 42u^5 + 76u^3 + 15u^2 + 70u$
41.  $2x - 1/2\sqrt{x}$
43.  $3x^2 - 2 + 1/\sqrt{x}$
45.  $-1$
47.  $317.44$
49.  $5$
51.  $x^3/3 + C$
53.  $x^{n+1}/(n+1) + C$
55.  $(kf)'(x) = 2akx + kb = kf'(x)$
57.  $\frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2 = \text{surface area}$
59.  $k$  is the conversion factor from miles to kilometers.

## 1.5 Products and Quotients

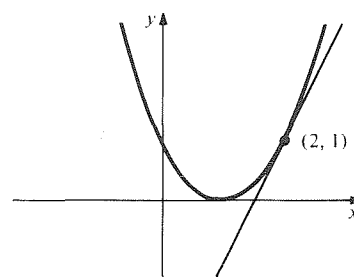
1.  $3x^2 + 16x + 2$
5.  $3x^2 + 2x - 1$
9.  $3x^2$
13.  $5x^{3/2}/2$
17.  $(-x^2 + 4x + 3)/(x^2 + 3)^2$
19.  $(4x^9 + 7x^6 + x^4 - 2x)/(x^3 + 1)^2$
21.  $-8x/(x^2 - 2)^2$
23.  $-2/x^3 + (-x^2 + 1)/(x^2 + 1)^2$
25.  $(-6r^2 - 16)/r^9$
27.  $2s/(1 - s)^3$
29.  $(-8x^3 - 8x)/(x^4 + 2x^2)^2$
31.  $-4/x^5$
33.  $-2/(x + 1)^3$
35.  $s^2(8s^5 + 5s^2 + 3)$
37.  $8y + 6 + 1/(y + 1)^2$
39.  $1/\sqrt{x}(\sqrt{x} + 1)^2$
41.  $15x^{3/2}/2 + 2x$
43.  $(2 + \sqrt{x})/2(1 + \sqrt{x})^2$
45.  $-3/\sqrt{2x}(1 + 3\sqrt{x})^2$
47. Let  $h(x) = 1/g(x)$  and differentiate  $1/h(x)$  by the reciprocal rule.
49.  $-1/4\sqrt{2}$
51.  $32x^7 + 48x^5 - 20x^4 - 52x^3 - 52x + 13$
53.  $24x^5 + 3x^2 - 26x + 2$
55.  $(-8x^7 - 32x^5 + 20x^4 - 26x^3 - 13)/(4x^5 - 13x)^2$
57.  $P(x) = x^2$  is an example.

## 1.6 The Linear Approximation and Tangent Lines

1.  $y = -2x + 2$



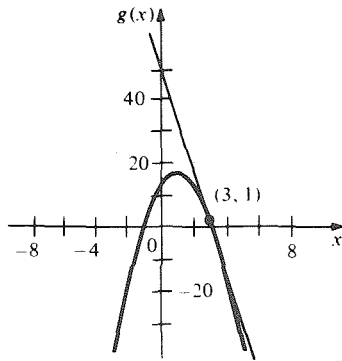
3.  $y = 2x - 3$



5.  $y = -(21/2)x$
7.  $y = (-25/2)x + (17/2)$
9.  $x = -1$
11.  $x = 16$
13. 4.08; 4.0804 is exact.
15. 24.99; 24.990001 is exact.
17. 4.002; 4.001999 on a calculator.
19. 3.99; 3.989987 on a calculator.
21. 74.52
23. 63.52
25.  $0.24\pi$
27.  $0.40\pi$
29. 60.966
31. 0.0822
33.  $y = 12x - 8$
35. 1.02
37.  $-4.9976$
39. 1,153,433.6
41.  $-14.267$ ;  $-0.98$ ;  $-36.00$

## A.12 Chapter 1 Answers

43.  $g'(3) = -16$ ; the graph lies below the tangent line.



45. (a)  $16 + 32(\Delta x)$   
 (b) Smaller  
 (c)  $[1.730, 2.247]$   
 47.  $1 - x$  is the linear approximation of  $1/(1+x)$  near 0.  
 49.  $\frac{3600}{60+x} \approx 60 - x + \frac{x^2}{60}$

## Review Exercises for Chapter 1

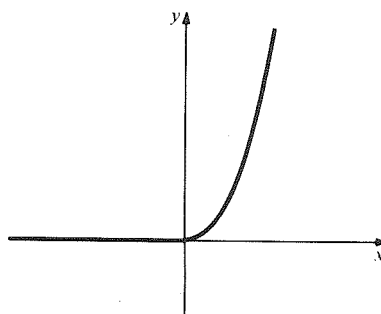
1.  $2x$
3.  $3x^2$
5. 2
7.  $2s + 2$
9.  $-50x^4 + 24x^2$
11.  $4x^3$
13.  $9x^2 - 1/\sqrt{x}$
15.  $50x^{49} - 1/x^2$
17.  $-4x/(x^2 - 1)^2$
19.  $-(5x^2 + 2)/2x^{3/2}(x^2 + 2)^2$
21.  $(5s^2 + 8s^{3/2} + 6s + 8\sqrt{s} + 1)/2\sqrt{s}$
23.  $(4\pi r + 3\pi r^{3/2})/2(1 + \sqrt{r})^2$
25.  $-6t/(3t^2 + 2)^2$
27.  $2\sqrt{2}p/(p^2 + 1)^2$
29.  $-(2\sqrt{x} - 1)/2\sqrt{x}(x - \sqrt{x})^2$
31. 2
33. 3
35. -192
37. 11
39. 6
41.  $4x^3 + 6x$
43.  $3/5$
45. 0
47. 1; 0; does not exist; does not exist; 1, 1, 2
49. 17
51.  $1 - 1/2\sqrt{x}$
53. -13
55. 0
57. 0
59.  $t = 2, 1/3$
61. 1 meter/second.
63. 1.072
65. 2.000025
67. 1.0045
69. 27.00
71.  $y = -6x$
73.  $y = (216/361)x - (413/361)$
75.  $0.16\pi$ , the exact value is calculated as  $(0.1608013 \dots)\pi$  meters<sup>3</sup>.
77.  $5/2$
79.  $-x$
81.  $dV/dr = 9r^2$  which is  $9/14$  of  $A$ .
83. (a)  $dz/dy = 4y + 3$ ;  $dy/dx = 5$ .  
 (b)  $z = 50x^2 + 35x + 5$ ;  $dz/dx = 100x + 35$   
 (c)  $(dz/dy) \cdot (dy/dx) = 100x + 35 = \frac{dz}{dx}$   
 (d)  $x = (y/5) - (1/5)$ ,  $dx/dy = 1/5$   
 (e)  $dx/dy = 1/(dy/dx)$
85.  $y = (2\sqrt{2} - 2)x$
87. The focal point is  $(0, 1/4a)$ .
89. (a) Use  $x^n x^m = x^{n+m}$ .  
 (b) Apply (a) to the numerator and denominator.  
 (c) Use the quotient rule. (If  $\deg f = 0$ , then  $f(x)$  might be constant, in which case,  $\deg f'$  is not  $-1$ .)
91. (a) Expand  $f'g - fg'$  and equate coefficients of  $1, x, x^2, \dots$  to zero.  
 (b) Apply (a) to  $F - G$ .

## Chapter 2 Answers

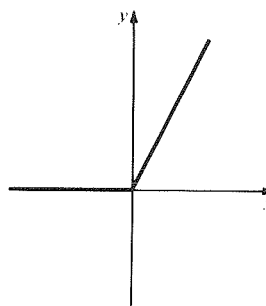
### 2.1 Rates of Change and the Second Derivative

1.  $y = 5x - 19$
3.  $y = \frac{1}{2}x - \frac{1}{2}$
5.  $\Delta P/\Delta t = 0.6$  cents/year,  $P = 5$  when  $t = 1987$ ;  
when  $t = 1991$ ,  $P = 7.4$
7.  $v = 9.8t + 3$ ;  $v = 150$  meters/second.
9.  $-60$
11.  $11/2$
13.  $\pi r^2$
15.  $69.6$
17.  $53$
19.  $400$  people/day
21.  $4\pi r^2$ , the surface area of a sphere
23.  $t = \pm \sqrt{-c/3}$ ; no
25.  $1.00106, 0.043$
27.  $12x^2 - 6$
29.  $10/(x+2)^3$
31.  $168r^6 - 336r^5$
33.  $2$
35.  $20x^3 + 84x^2$
37.  $2/(x-1)^3$
39.  $4(3t^2 + 1)/(t^2 - 1)^3$
41.  $3$  meters/second;  $0$  meters/second<sup>2</sup>
43.  $0$  meters/second;  $16$  meters/second<sup>2</sup>
45.  $-2$  meters/second;  $0$  meters/second<sup>2</sup>
47. (a)  $29.4$  meters/second downward;  $-9.8$  meters/second<sup>2</sup>.  
(b)  $-20.8$  meters/second.
49.  $10$  dollars/worker
51.  $94$  dollars/worker day
53.  $[25 - 0.04x - (4 + 0.04x - 0.016x^3 + 0.00004x^4)/(1 + 0.002x^3)^2]$  dollars/boot.
55. Gas mileage (in miles/gallon).
57. Price of fuel (in dollars/gallon).
59. The first term is the rate of cost increase due to the change in price and the second term is the rate of cost increase due to the change in consumption rate.
61.  $-1.6, -1.996, -1.999996$ . (The derivative is  $-2$ )
63. (a)  $0; 0$   
(b)  $30; 30$   
(c)  $0; 0$   
(d)  $-12; -12$
65.  $(10t^4 + 4t^3 + 12t^2 + 22t - 3)$  cm<sup>2</sup>/second.
67. (a)  $t > 6/10$   
(b)  $-13/2$   
(c)  $t = 1/6$
69.  $a = 0$
71. (a) Accelerating when  $t > 0$ ; decelerating when  $t < 0$ .  
(b) no
73. (a)  $V = 4000 - 350t$   
(b)  $-350$

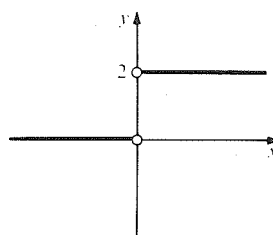
75. (a)



$$(b) f'(x) = \begin{cases} 0 & x \leq 0 \\ 2x & x \geq 0 \end{cases}$$



$$(c) f''(x) = \begin{cases} 0 & x < 0 \\ 2 & x > 0 \end{cases}$$

There is no second derivative at  $x = 0$ .

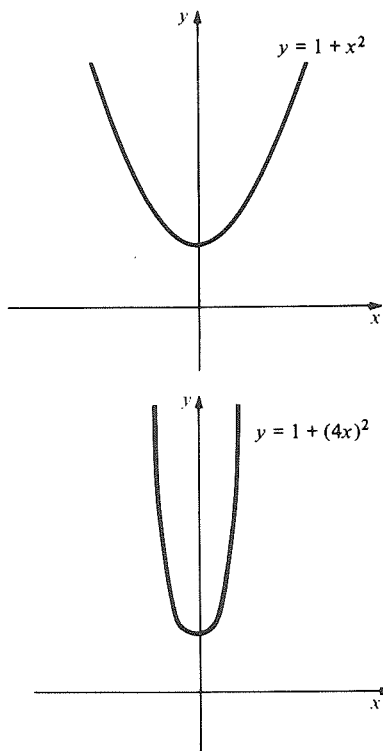
(d) The acceleration may have jumped as an engine was started.

### 2.2 The Chain Rule

1.  $4(x+3)^3$
3.  $100(x^3 + 10x)^{99}(3x^2 + 10)$
5.  $(x^2 + 8x)^2(7x^2 + 32x)$
7.  $(x^9 + 8)^2(29x^{10} + 54x^8 + 16x)$
9.  $(y+1)^2(y+2)(6y^2 + 26y + 26)$
11.  $(x+1)^2; x^2 + 1$
13.  $(x^3 - 2)^3; (x-2)^9$
15.  $\frac{1}{2} - \sqrt{3}/(1-x); 2/(1+2\sqrt{3}x)$

## A.14 Chapter 2 Answers

17.  $h(x) = f(g(x))$  where  $f(u) = \sqrt{u}$ ,  $g(x) = 4x^3 + 5x + 3$ .
19.  $h(u) = f(g(u))$  where  $f(v) = v^3$ ,  $g(u) = (1-u)/(1+u)$
21.  $4x(x^2 - 1)$
23. 1
25.  $6(x^2 - 6x + 1)^2(x - 3)$
27.  $-195x^4/(3 + 5x^5)^2$
29.  $8x(x^2 + 2)[(x^2 + 2)^2 + 1]$
31.  $-2x(x^2 + 3)^4(3x^2 + 4)/(x^2 + 4)^9$
33.  $5x(2x^3 + 1)/\sqrt{4x^5 + 5x^2}$
35.  $3x^2f(2x^2) + 4x^4f'(2x^2)$
37. (a)  $f(g(h(x)))$   
(b)  $f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$
39. 0.2;  $\approx 0.18$ ; as the circle grows larger, a given increase in diameter produces more area
41.  $1500 \text{ gm cm}^2/\text{sec}^3$
43.  $5\phi/\text{mile}$
45.  $-6250$
47.  $156(x + 1)^{11}$
49.  $392(x^4 + 10x^2 + 1)^{96}[391x^6 + 3915x^4 + 53x^2 + 9700x + 5]$
51. (a)  $(d/dx)f(cx) = cf'(cx)$   
(b) If the  $x$ -axis is compressed by a factor 4, the slopes are multiplied by 4.



53.  $\frac{d^2}{dx^2}(u^n) = nu^{n-2} \left[ u \frac{d^2u}{dx^2} + (n-1) \left( \frac{du}{dx} \right)^2 \right]$
55.  $f'(x) = 1/g'(f(x))$
57.  $(f \circ g)''(x) = f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$

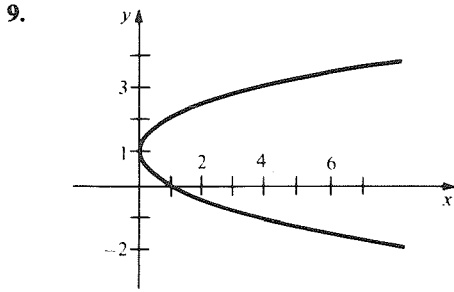
59. (a) Use the product rule  
(b) Use the quotient rule  
(c) Use the power of a function rule
- (d)  $n_1 \frac{f'_1(x)}{f_1(x)} + n_2 \frac{f'_2(x)}{f_2(x)} + \cdots + n_k \frac{f'_k(x)}{f_k(x)}$   
$$\frac{(x^2 + 3)(x + 4)^8(x + 7)^9}{(x^4 + 3)^{17}(x^4 + 2x + 1)^5} \times$$
- (e) 
$$\left[ \frac{2x}{x^2 + 3} + \frac{8}{x + 4} + \frac{9}{x + 7} - \frac{68x^3}{x^4 + 3} - \frac{5(4x^3 + 2)}{x^4 + 2x + 1} \right]$$

## 2.3 Fractional Powers and Implicit Differentiation

1.  $5/(4x^{7/8})$
3.  $2/x^{3/4} + 2/(3x^{5/3})$
5.  $2/x^{1/3} - 5/(2\sqrt{5x})$
7.  $(7/3)x^{4/3} + (10/3)x^{7/3}$
9.  $35x^4/[9(x^5 + 1)^{2/9}]$
11.  $-1/(2x^{3/2})$
13.  $-2x/[(x^2 + 1)^{1/2}(x^2 - 1)^{3/2}]$
15.  $(x + 3)/\sqrt{(x + 3)^2 - 4}$
17.  $(3 - x - 5x^3)/[2\sqrt{x}(3 + x + x^3)^2]$
19.  $1/[2\sqrt{x}(1 + \sqrt{x})^2]$
21.  $(2 - 5x^2)/[3x^{2/3}(x^2 + 2)^2]$
23.  $(18x^2 + 2x - 24x^{9/2} - 16x^{7/2} - 12x^{5/2} - 1)/[4\sqrt{x}(\sqrt{x} + 2x^3)^{3/2}\sqrt{6x^2 + 2x + 1}]$
25.  $7x/[4(x^2 + 5)^{1/8}]$
27.  $\sqrt[3]{7}/3$
29.  $(3/11)x^{-8/11} - (1/5)x^{-4/5}$
31.  $-(7y + 2)/[8y^{7/8}(y - 2)^2]$
33.  $-1/[2\sqrt{x}(\sqrt{x} - 1)^{3/2}(\sqrt{x} + 1)^{1/2}]$
35. 0
37. (a)  $-4x^3/(2y + 1)$       39. (a)  $\sqrt{2}/4$   
(b)  $-4/3$       (b)  $-\sqrt{2}/4$   
(c)  $-4x^3/(2y + 1)$       41.  $y = -x + 2$
43.  $(-1/4)x^{-3/2} + (2/9)x^{-4/3}$
45.  $y = -\sqrt{3}x + 2$       49. 1.9990625
47.  $x^3/(1 - x^4)^{3/4}$       51. 8.9955
53.  $[32(2 + x^{1/3})^3/x^{2/3}] \text{ kg/unit distance.}$
55. 2.4
57.  $-2\sqrt{2}/27 \text{ seconds/pound.}$
59. (a)  $y = 50x/(x - 50)$   
(b)  $-2500/(x - 50)^2$   
(c)  $(0, 0)$  or  $(100, 100)$

## 2.4 Related Rates and Parametric Curves

1.  $dy/dt = (x/y)(dx/dt)$
3.  $dy/dt = (dx/dt)/(2y + 3y^2)$
5.  $dy/dt = (dx/dt)/(1 - 2y)$
7.  $dy/dt = \sqrt{y/x}(dx/dt)$



11.  $y = x^3$  for  $x \geq 0$

13.  $y = (15x - 125)/2$

15.  $y = \frac{10}{\sqrt{3} - 1} + \frac{\sqrt{159}(13 - 6\sqrt{3})}{76(-6 + 4\sqrt{3})}(x - \sqrt{159})$

17.  $dy/dt = -(dx/dt)/16$

19.  $-41/8$

21.  $-125/8\pi$  cm/sec.

23. 2.55 inches/minute.

25. (a) By the distance formula,

$$\sqrt{x^2 + y^2} = 2\sqrt{x^2 + (y - 1)^2}$$

which yields  $3x^2 + 3(y - (4/3))^2 = 4/3$ , a circle.

(b) 0

(c)  $(\sqrt{2}/3, (-\sqrt{2} + 4)/3)$ ,  $(-\sqrt{2}/3, (\sqrt{2} + 4)/3)$ 

27.  $-7/3$

29.  $-1$

31. (a) For a horizontal line,  $dy/dt = 0$ ,  $dx/dt \neq 0$ .  
For a vertical line,  $dx/dt = 0$ ,  $dy/dt \neq 0$ .(b) Horizontal tangents at  $t = \pm\sqrt{1/3}$ . Vertical tangent at  $t = 0$ .

33. Use  $\frac{dh}{dt} \bigg/ \frac{dV}{dt} = \frac{dh}{dV}$

## 2.5 Antiderivatives

1.  $x^2/2 + 2x + C$

3.  $s^4/4 + s^3 + s^2 + C$

5.  $-1/2t^2 + C$

7.  $2x^{5/2}/5 - 2x^{3/2}/3 + C$

9. 6

11.  $(1 + 4\sqrt{2})/24$

13.  $(3/2)x^2 + C$

15.  $-x^{-1} - (1/2)x^{-2} + C$

17.  $2(x + 1)^{3/2}/3 + C$

19.  $2(t + 1)^{5/2}/5 + C$

21.  $v = 9.8t + 1$ ;  $x = 4.9t^2 + t + 2$

23.  $v = 9.8t - 2$ ;  $x = 4.9t^2 - 2t$

25. No.

27. Use the sum rule for derivatives.

29.  $x^3/3 + 3x^2/2 + 2x + C$

31.  $t^3 + t^2 + t + C$

33.  $-1/[8(8t + 1)] + C$

35.  $-1/[6(3b + 2)^8] + C$

37.  $-1/3x^3 + x^5/5 + C$

39.  $2x^{5/2}/5 + 6\sqrt{x} + C$

41.  $(1/4)x^4 + (3/2)x^2 + C$

43.  $-1/(t + 1) + C$

45.  $(8x + 3)^{3/2}/12 + C$

47.  $-4(8 - 3x)^{5/2}/3 + C$

49.  $x + 6\sqrt{x - 1} + C$

51.  $x^3/3 + 3x^2/2 + 9x + C$

53. 4.6 seconds

55.  $\approx 3195$

57. 4 seconds

59. (a)  $-6x^2/(x^3 - 1)^2$

(b)  $(x^3 + 1)/(x^3 - 1) + C$

61. (a)  $80(x^4 + 1)^{19}x^3$

(b)  $(x^4 + 1)^{20}/80 + 9x^{5/3}/5 + C$

63.  $-(3/4)x^{-4} + (2/3)x^{-3} + x^{-1} + C$

65.  $(1/4)x^4 + x^3 + 2x + 1$

67. (a)  $\int [f(x)]^n f'(x) dx = [f(x)]^{n+1}/(n+1) + C$

(b)  $-1/6(x^3 + 4)^2 + C$

## Review Exercises for Chapter 2

1.  $18(6x + 1)^2$

3.  $10(x^3 + x^2 - 1)(3x^2 + 2x)$

5.  $-6/x^2$

7.  $-2/(x - 1)^2$

9.  $(-2x^3 - 54x)(x^2 + 1)^{12}/(x^2 - 1)^{15}$

11.  $3(x^2 + x/4 - 3/8)^2(2x + 1/4)$

13.  $(x^3 + 12x^2 - 6x - 8)/(x + 4)^3$

15.  $24x^2 - 8x - 4$

17.  $y = -1 - \sqrt[3]{2}(x - 1)/6$

19.  $y = -5184 - 3240x$

21.  $(5/3)x^{2/3}$

23.  $(3\sqrt{x} + x^{5/2})/[2(1 + x^2)^{3/2}]$

25.  $3\sqrt{x}/(1 - x^{3/2})^2$

27.  $4/\sqrt{x}(1 + \sqrt{x})^2 + 3(1 - x + \sqrt{x})/(1 - \sqrt{x})^2$

29.  $f'(x) = (-x^2 + 2ax + c + 2ab)/(x^2 + 2bx + c)^2$

$$f''(x) = [2x^3 - 6ax^2 - (6c + 12ab)x + 2ac - 8ab^2 - 4bc]/(x^2 + 2bx + c)^3$$

31.  $x'(t) = A/(1 - t)^2 + 2Bt/(1 - t^2)^2 + 3Ct^2/(1 - t^3)^2$

$$x''(t) = 2A/(1 - t)^3 + (2B + 6Bt^2)/(1 - t^2)^3 + (6Ct + 12Ct^4)/(1 - t^3)^3$$

33.  $h'(r) = 13r^{12} - 4\sqrt{2}r^3 - (-r^2 + 3)/(r^2 + 3)^2$

$$h''(r) = 156r^{11} - 12\sqrt{2}r^2 - (2r^3 - 18r)/(r^2 + 3)^3$$

35.  $f'(x) = 3(x - 1)^2g(x) + (x - 1)^3g'(x)$

$$f''(x) = 6(x - 1)g(x) + 6(x - 1)^2g'(x) + (x - 1)^3g''(x)$$

37.  $h'(x) = 2(x - 2)^3(3x^2 - 2x + 4)$

$$h''(x) = 2(x - 2)^2(15x^2 - 20x + 16)$$

39.  $g'(t) = (2t^7 - 3t^6 - 5t^4 + 12t^3)/(t^3 - 1)^2$

$$g''(t) = (2t^9 - 4t^6 - 18t^5 + 20t^3 - 36t^2)/(t^3 - 1)^3$$

41. If  $V = \frac{4}{3}\pi r^3$ ,  $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ . You are given

$$\frac{dV}{dt} = k \cdot 4\pi r^2.$$

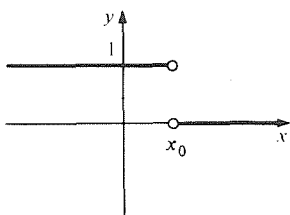
**A.16 Chapter 2 Answers**

43. (a)  $3/4$  millibars per degree.  
(b)  $-12^\circ\text{C}$
45. 0
47. 32.01 miles per hour.
49.  $dA/dx = 44x$ ;  $d^2A/dx^2 = 44$ .
51.  $dA/dy = [120 - (25/2)\pi]y$ ;  $\frac{d^2A}{dy^2} = 120 - \left(\frac{25}{2}\right)\pi$
53.  $dA/dP = 44P/(25 + \sqrt{13})^2$ ;  $dP/dx = 25 + \sqrt{13}$
55.  $dA/dP = [120 - (25/2)\pi]P/[5\pi/2 + 32]^2$ ;  
 $dP/dy = (5\pi/2) + 32$
57. (a)  $[5 - (0.02)x]$  dollars per case.  
(b) \$3.32  
(c) Marginal cost is a decreasing function of  $x$ .  
(d)  $x \geq 504$
59.  $y = 0$
61.  $y = \frac{1}{19} + \frac{216}{361}(x - 2)$
63.  $-1/2$
65.  $y = (3 + \sqrt[3]{2}) + \frac{(1 + 3\sqrt[3]{4})4\sqrt{2}}{(15\sqrt{2} + 2)3\sqrt[3]{4}} \left(x - \frac{9}{2} - \sqrt{2}\right)$
67. (a) 3.00407407  
(b)  $-3.979375$
69. (a)  $20\Delta x$   
(b) 0.42
71.  $f''(x)g(x) + 2f'(x)g'(x) + f(x)g''(x)$
73. Each side gives  $mn f(x)^{mn-1}f'(x)$ .
75.  $10x + C$
77.  $x^4 + x^3 + x^2 + x + C$
79.  $10x^{7/5}/21 + C$
81.  $1/x + 1/x^2 + 1/x^3 + 1/x^4 + C$
83.  $x^3/3 + 2x^{3/2}/3 + C$
85.  $2x^{5/2}/5 + 2\sqrt{x} + C$
87.  $2x^{9/2}/9 + 2x^{13/2}/13 + 2\sqrt{x} + C$
89.  $2(x-1)^{3/2}/3 + C$
91.  $-1/(x-1) + C$
93.  $2(x-1)^{3/2}/3 - 2(x-2)^{7/2}/7 + C$
95.  $f'(x) = \frac{4x^{1/2} + x - 2x^{5/6}}{12x^{7/6}(x^{1/2} - x^{1/3} + 1)^{3/2}}$ ;  
 $\int f'(x)dx = x^{1/3}/(x^{1/2} - x^{1/3} + 1)^{1/2} + C$
97.  $f'(x) = 1/2\sqrt{x} - 1/(x+1)^{3/2}\sqrt{x-1}$ ;  
 $\int f'(x)dx = \sqrt{x} - \sqrt{(x-1)/(x+1)} + C$
99.  $f'(x) = 1/2x^{3/4}(\sqrt[4]{x} + 1)^2$ ;  
 $\int f'(x)dx = (\sqrt[4]{x} - 1)/(\sqrt[4]{x} + 1) + C$
101.  $f'(x) = -2x/(x^2 - 1)^{3/2}\sqrt{x^2 + 1}$ ;  
 $\int f'(x)dx = [(x^2 + 1)/(x^2 - 1)]^{1/2} + C$
103. (a) Replacing  $x$  by  $-x$  does not change the equation, so the graph is symmetric to the  $y$ -axis. Similar argument for  $x$ -axis.  
(b)  $-2/11$   
(c)  $y = -2x/11 + 15/11$
105. 84 pounds per second.
107. (a)  $f''(x) = n(n-1)x^{n-2}$ ;  
 $f'''(x) = n(n-1)(n-2)x^{n-3}$   
(b)  $\frac{d^r x^n}{dx^r} = n(n-1)(n-2)\dots(n-r+1)x^{n-r}$   
(c)  $[f(x)g(x)h(x)]' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$
109. By Exercise 108,  $\frac{d^{k-1}}{dx^{k-1}} r(x) = C_1$ . Conclude  
 $\frac{d^{k-2}}{dx^{k-2}} r(x) = C_1x + C_2$  and repeat.

## Chapter 3 Answers

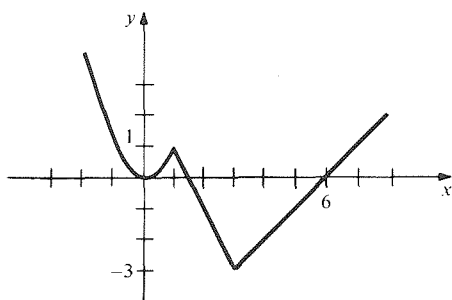
### 3.1 Continuity and the Intermediate Value Theorem

1. (a)  $(-\infty, -1)$ ,  $(-1, 1)$ ,  $(1, \infty)$ ; (b) all  $x$ ;  
(c)  $(-\infty, -1)$ ,  $(-1, \infty)$
3. Use the rational function rule.
5. Use the rational function rule.
7.  $(-\infty, \infty)$
9. Since  $f(x)$  is defined everywhere but at  $x_0 = \pm 1$ ,  $f(x)$  is continuous by the rational function rule.
11.  $\lim_{x \rightarrow 0} f(x)$  does not exist.
13.  $(f+g)(x_0) = f(x_0) + g(x_0) =$   
 $\lim_{x \rightarrow x_0} f(x) + \lim_{x \rightarrow x_0} g(x) = \lim_{x \rightarrow x_0} [f(x) + g(x)].$
15.  $x < -2\sqrt{2}$ ,  $x > 2\sqrt{2}$
17. Let  $f(s) = -s^5 + s^2 - 2s + 6$ . Note that  $f(2) = -26$ ,  $f(-2) = 46$  and use the intermediate value theorem.
19. Consider  $f(-1) = 3$ ,  $f(0) = -1$ ,  $f(1) = 3$  and use the intermediate value theorem.
21. Negative on  $(-\infty, -\sqrt{2})$  and  $(1, \sqrt{2})$ , positive on  $(-\sqrt{2}, 1)$  and  $(\sqrt{2}, +\infty)$
23.  $\approx 1.34$
25. Use the second version of the intermediate value theorem.
27. No,  $\lim_{x \rightarrow x_0} f(x)$  does not exist.



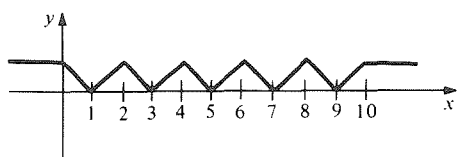
29.  $f(2) = 4$

31. One possibility is  $f(x) = -5x/2 + 9/2$ .



33.  $f(1) = 2$

35.



37.  $f(x)$  is not continuous on  $[0, 2]$ .
39. 2.22
41. -1.194
43. To increase accuracy by one decimal place, accuracy must increase ten times.
45.  $|10f(b)/[f(b) - f(a)]|$  computed to the nearest integer marks the new division point if  $[a, b]$  is divided into ten equal parts. (Other answers are possible.)
47. If  $f$  is continuous at  $x_0$  and  $f(x_0) > 0$  then  $f(x) > 0$  for  $x$  in some interval about  $x_0$ .
49. In step 2, write  $f(x)/x^n = 1 + a_{n-1}x^{-1} + \dots + a_0x^{-n}$  and  $|a_{n-1}x^{-1} + \dots + a_0x^{-n}| \leq |a_{n-1}x^{-1}| + \dots + |a_0x^{-n}| \leq |a_{n-1}||x^{-1}| + \dots + |a_0||x^{-n}| = (|a_{n-1}| + \dots + |a_0|)|x^{-1}| < 1/2$ , so  $f(x)/x^n > 1/2$ .

### 3.2 Increasing and Decreasing Functions

1. Use the interval  $(2, 4)$  and the definition of increasing.
3. Verify the definition of increasing function.
5.  $f$  changes from **negative to positive** at  $x_0 = 1/2$ .
7.  $f$  changes from negative to positive at  $x_0 = 0$ .
9. Increasing
11. Neither
13. Right
15. January, February, March;  $t = 3$  is a minimum point; inflation got worse after April 1.
17. Increasing on  $(0, \infty)$ ; decreasing on  $(-\infty, 0)$
19. Increasing on  $(-\infty, 1)$ ,  $(2, \infty)$ ; decreasing on  $(1, 2)$ .
21. (a) 3 (b) 4  
(c) 1 (d) 2  
(e) 5
- 23.

| Function \ Question      | (a)     | (b)                             | (c)            | (d)            | (e)                         | (f)                 |
|--------------------------|---------|---------------------------------|----------------|----------------|-----------------------------|---------------------|
| (1) increasing           | nowhere | $(-\infty, 0)$<br>$(0, \infty)$ | $(0, \infty)$  | $(0, \infty)$  | $(-\infty, -1)$<br>$(0, 1)$ | $(-\infty, \infty)$ |
| (2) decreasing           | nowhere | nowhere                         | $(-\infty, 0)$ | $(-\infty, 0)$ | $(-1, 0)$<br>$(1, \infty)$  | nowhere             |
| (3) local min-max points | none    | none                            | none           | 0 (min)        | 0 (min)<br>$\pm 1$ (max)    | none                |
| (4) sign changes         | 0       | 0                               | 1              | nowhere        | $\pm 2$                     | 1                   |

25.  $x_1, x_3, x_5$  are minima;  $x_2, x_4$  are maxima;  $x_6$  is neither.
27.  $x = 0$ , a local minimum.
29.  $x = 0$ , a local minimum;  $x = -2/3$ , a local maximum.
31.  $y = -1$ , a local minimum;  $y = 1$ , a local maximum.
33.  $r = -1, 0, 1$  are local minima;  $r = \pm 1/\sqrt{2}$  are local maxima.
35. Increasing at  $-3, -36$ ; decreasing at  $1, 3/4, 25$ .

## A.18 Chapter 3 Answers

37.  $m = -2$ , positive to negative;  $m = 0$ , no sign change;  $m = 2$ , negative to positive.  
 39.  $x_0 = -2$ , positive to negative;  $x_0 = -1$ , negative to positive.  
 41.  $f$  does not change sign.  
 43. Positive to negative.  
 45. Increasing on  $(-\infty, -\sqrt{5/6})$ ,  $(\sqrt{5/6}, \infty)$ ; decreasing on  $(-\sqrt{5/6}, \sqrt{5/6})$ .  
 47. Increasing on  $(-\infty, 3 - \sqrt{10})$ ,  $(3 + \sqrt{10}, \infty)$ ; decreasing on  $(3 - \sqrt{10}, 3 + \sqrt{10})$ .  
 49.  $f(x)$  is of the form  $a(x^2 - 4x + 3)$  where  $a > 0$ .  
 51. (a)  $\lim_{x \rightarrow 1} f(x) = 3/2$ ,  $\lim_{x \rightarrow -1} f(x)$  does not exist.  
 (b) Show  $f'(x) = x(x^3 - 3x + 2)/(x^2 - 1)^2 > 0$  on  $(-\infty, -2)$ .  
 (c)  $f(a) < f(-2)$  if  $a < -2$  or  $-2 < a < -1$ .  
 (d) If  $a > -1$ , then  $(a^3 - 1)/(a^2 - 1) \geq 1$ .  
 53. For  $x < x_0 < y$ ,  $f(x) < f(x_0) < f(y)$  and  $g(x) < g(x_0) < g(y)$  implies  $f(x) + g(x) < f(x_0) + g(x_0) < f(y) + g(y)$ .  
 55. If  $a_1 > 0$ . If  $a_1 = 0 = a_2$  and  $a_3 > 0$ . Etc.  
 57. (a)  $g(x)h(x)$  is increasing when  $g'/g + h'/h > 0$ .  
 $g(x)h(x)$  is decreasing when  $g'/g + h'/h < 0$ .  
 (b)  $g(x)/h(x)$  is increasing when  $g'/g - h'/h > 0$ .  $g(x)/h(x)$  is decreasing when  $g'/g - h'/h < 0$ .  
 59.  $f(x) = k(x^3/3 - x)$ ,  $k < 0$   
 61.  $2a^2 = b^2$

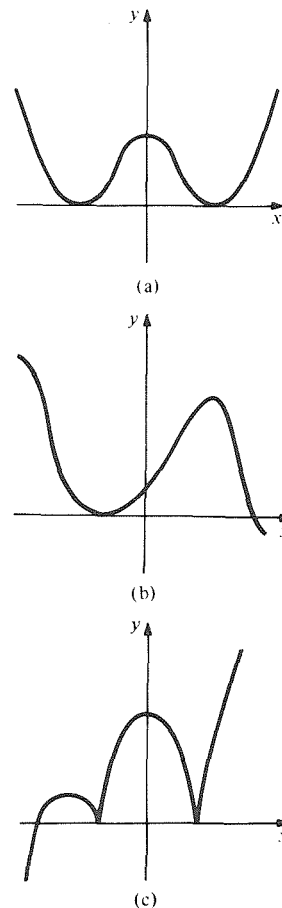
on  $(-2, 2/3)$ , concave up on  $(-2/3, \infty)$ , concave down on  $(-\infty, -2/3)$ .

31. Inflection point at  $x = 0$  for odd  $n$ ,  $n > 2$ .  
 33.  $f(x) = kx^4/12 - kx^3/2 + kx^2 + dx + e$  where  $k \neq 0$ ,  $d$  and  $e$  arbitrary.  
 35. (a)  $f''(x_0) < 0$  ( $f''(x_0) > 0$ ) makes the linear approximation greater (less).  
 (b) the approximation is less for  $x > 0$ .  
 37. (a)  $2(x + 1)$ ,  $-x$ ,  $2(x - 1)$   
 (b)

| $\Delta x \backslash x_0$ | -1               | 0                | 1                |
|---------------------------|------------------|------------------|------------------|
| 1                         | 0.2              | 0, -1,           | 6.2              |
| -1                        | -6, -2           | 0, 1             | 0, -2            |
| 0.17                      | 0.171, 0.2       | -0.099, -0.1     | 0.231, 0.2       |
| -0.1                      | -0.231, -0.2     | 0.099, 0.1       | -0.171, -0.2     |
| 0.01                      | 0.019701, 0.02   | -0.009999, -0.01 | 0.020301, 0.02   |
| -0.01                     | -0.020301, -0.02 | 0.009999, 0.01   | -0.019701, -0.02 |

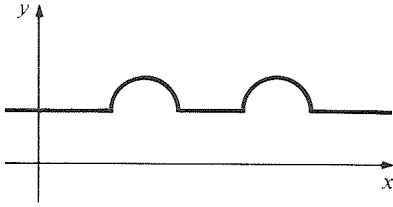
This table shows  $f(x_0 + \Delta x)$ , followed by its linear approximation.  $e(x) < 0$  when  $f''(x_0) < 0$ ,  $e(x) > 0$  when  $f''(x_0) > 0$ .  $e(x)$  is the same sign as  $\Delta x$  when  $f''(x_0) = 0$ .

39. (a)  $I = E/2R$   
 (b)  $P = E^2/4R$   
 41. 28 feet.  
 43.



## 3.3 The Second Derivative and Concavity

1.  $x = 0$  is a local minimum.  
 3.  $x = -\sqrt[4]{1/30}$  is a local maximum;  $x = \sqrt[4]{1/30}$  is a local minimum.  
 5.  $x = 0$  is a local minimum.  
 7.  $s = -1$  is a local minimum;  $s = 1$  is a local maximum.  
 9. Concave up everywhere.  
 11. Concave up everywhere.  
 13. Concave up on  $(1, \infty)$ , concave down on  $(-\infty, 1)$ .  
 15. Concave up on  $(-4/3, \infty)$ , concave down on  $(-\infty, -4/3)$ .  
 17.  $x = 0$   
 19.  $x = 0$   
 21. None  
 23.  $x = \pm 1/\sqrt{3}$   
 25. (a) Maximum  
 (b) Inflection point  
 (c) None  
 (d) Maximum  
 (e) Maximum  
 (f) Inflection point  
 (g) Inflection point  
 (h) Minimum  
 27.  $x = 0$  is a local minimum, increasing on  $(0, \infty)$ , decreasing on  $(-\infty, 0)$ , concave up on  $(-\infty, \infty)$ .  
 29.  $x = -2$  is a local maximum,  $x = 2/3$  is a local minimum,  $x = -2/3$  is an inflection point, increasing on  $(-\infty, -2)$ , and  $(2/3, \infty)$ , decreasing



(d)

45. Not necessarily.

47. If  $h'' = f''$  changes sign from negative to positive at  $x_0$ , then  $h'(x)$  is decreasing to the left of  $x_0$  and increasing to the right. Since  $h'(x_0) = 0$ ,  $h'$  changes sign from negative to positive. Repeat to conclude the result.

### 3.4 Drawing Graphs

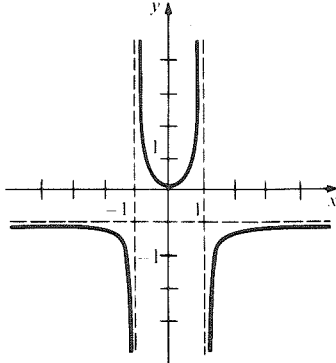
1. Odd

3. Neither

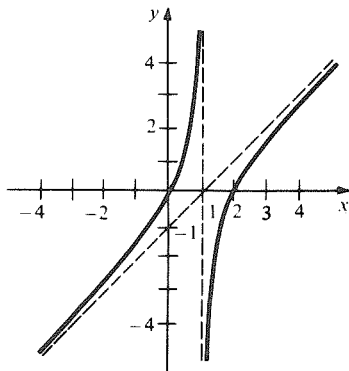
5. Near  $-1$ :  $x < -1$ ,  $f(x)$  is large and positive;  $x > -1$ ,  $f(x)$  is large and negative.

7. Near  $1$ :  $x < 1$ ,  $f(x)$  is large and negative;  $x > 1$ ,  $f(x)$  is large and positive. ( $x = -1$  is not a vertical asymptote).

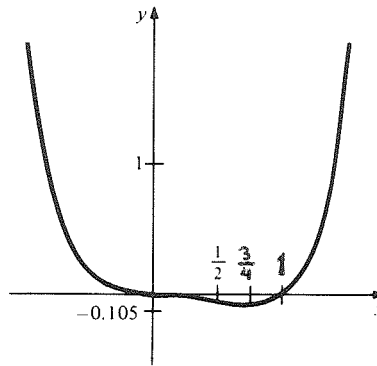
9.



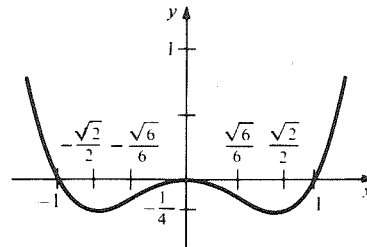
11.



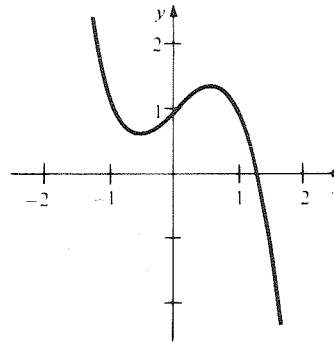
13.



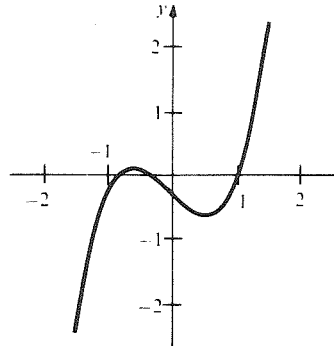
15.



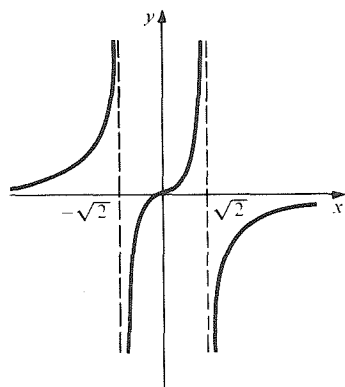
17.



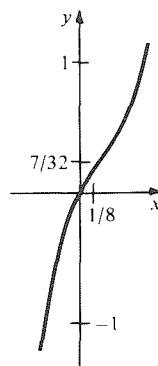
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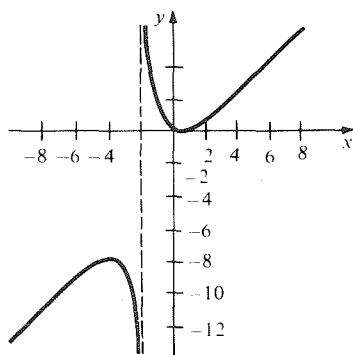
21.



29.



23.



31. (a)  $B$

(b)  $A$

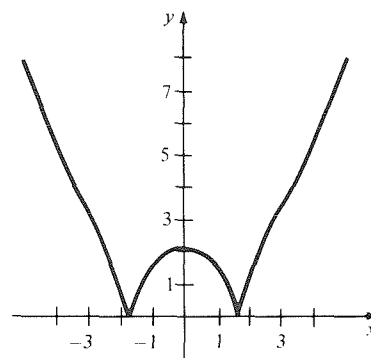
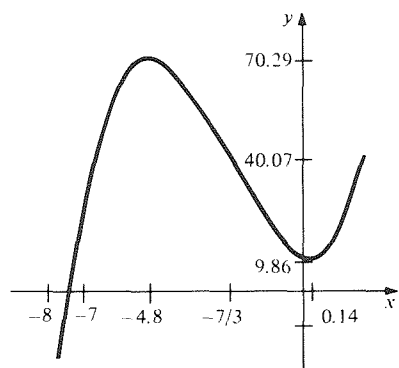
(c)  $D$

(d)  $C$

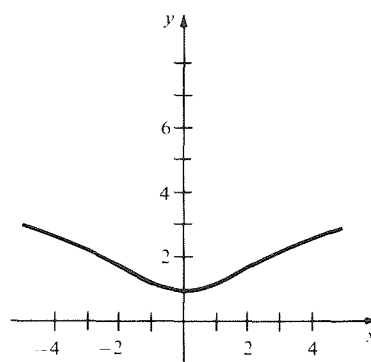
33. (a) Increasing on  $(-\sqrt{3}, 0)$  and  $(\sqrt{3}, \infty)$ ; decreasing on  $(-\infty, -\sqrt{3})$  and  $(0, \sqrt{3})$ .

(b)

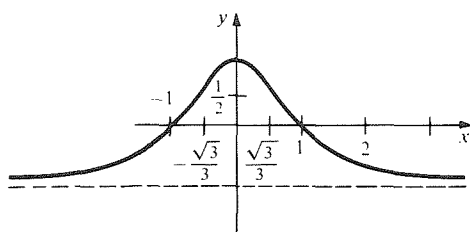
25.



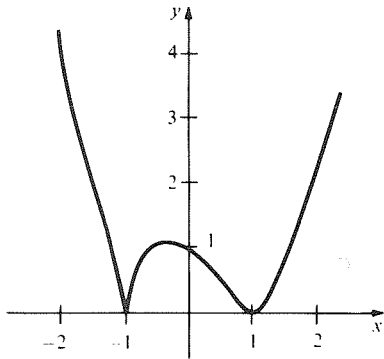
35.



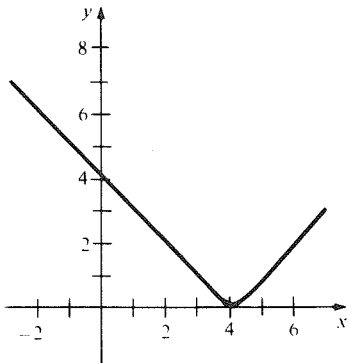
27.



37.

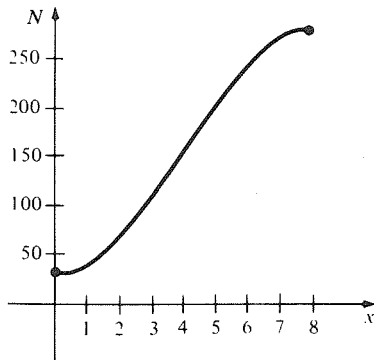


39.



41.  $f(x)$  is even if and only if  $f(x) - f(-x) = 0$ . When expanded, this shows that  $f(x)$  is even if the even powers of  $x$  have nonzero coefficients. Use a similar argument for odd functions.

43. (a)



(b)  $t = 4$

(c) The growth rate is zero.

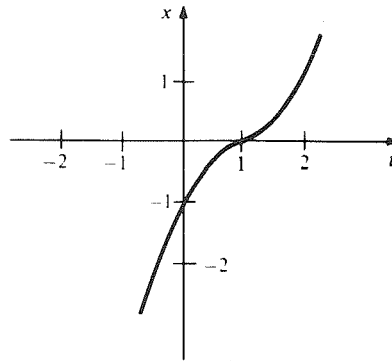
45.  $e(x) = \frac{1}{2}[f(x) + f(-x)]$ ,  $o(x) = \frac{1}{2}[f(x) - f(-x)]$

47. No; for example  $f(x) = 1$ .

49. Locate the inflection point of  $g$  at  $x = 0$  or just substitute.

51.  $x^3 + (8/3)x$ , type I

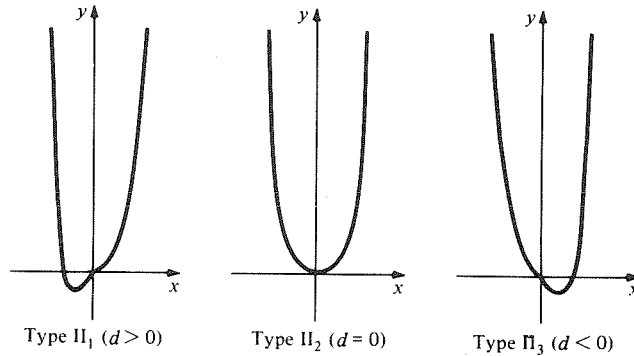
53. Velocity is 0, acceleration is infinite.



55. No critical points if  $ap > 0$ , 1 critical point if  $p = 0$ , 2 critical points if  $ap < 0$ .

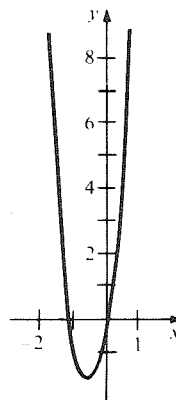
57. Substitute

59.



61. Eliminate  $x$  from the equations  $f'(x) = 0$ ,  $f''(x) = 0$ .

63. III<sub>1</sub>

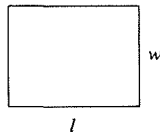


### 3.5 Maximum—Minimum Problems

1.  $(1, -1)$  is the minimum; no maximum.
3.  $(\sqrt{2}, 2\sqrt{2})$  is the minimum; no maximum.
5. No minimum. Maximum at  $x = -1$ .
7.  $-\sqrt{5/3}$  is a minimum,  $\sqrt{5/3}$  is a maximum.
9.  $(1971.4, 16\%)$  is a minimum;  $(1980, 34\%)$  is a maximum.

|       | Critical points           | Endpoints | Maxima                              | Minima                          |
|-------|---------------------------|-----------|-------------------------------------|---------------------------------|
| 11.   | 1/2                       | 0         | (0, 0)                              | (1/2, -1/4)                     |
| 13.   | $0, \sqrt{2}, -\sqrt{2}$  | -4, 2     | (-4, 199)                           | $(\sqrt{2}, 3), (-\sqrt{2}, 3)$ |
| 15.   | $1/2, -1/2$               | -10, 20   | $(1/2, 0.000005), (-1/2, 0.000005)$ | $(20, 0.00000119)$              |
| 17(a) | —                         | —         | —                                   | —                               |
| (b)   | —                         | 1/2       | —                                   | (1/2, -1/4)                     |
| (c)   | 3/2                       | 2         | —                                   | (3/2, -5/4)                     |
| (d)   | 3/2                       | 2         | —                                   | (3/2, -5/4)                     |
| (e)   | 3/2                       | —         | —                                   | (3/2, -5/4)                     |
| (f)   | 3/2                       | —         | —                                   | (3/2, -5/4)                     |
| (g)   | —                         | -1, 1     | (-1, 5)                             | (1, -1)                         |
| (h)   | 3/2                       | -8, 8     | (-8, 89)                            | (3/2, -5/4)                     |
| 19(a) | -1/7                      | -1, 1     | (1, 13)                             | (-1/7, 27/7)                    |
| (b)   | —                         | —         | —                                   | —                               |
| (c)   | -1/7                      | -4        | (-4, 108)                           | (-1/7, 27/7)                    |
| 21.   | 1                         | —         | —                                   | —                               |
| 23.   | $\sqrt{1/3}, -\sqrt{1/3}$ | -2, 3     | (3, 24)                             | (-2, -6)                        |
| 25.   | 0, -0.6                   | -10, 10   | (10, 999/101)                       | (-10, -1001/101)                |
| 27.   | -1, 1                     | -1, 6     | (-1, 1/2)                           | (1, -1/2)                       |
| 29.   | 0                         | —         | —                                   | —                               |

31. Minimize  $f(l) = 2(l + 1/l)$

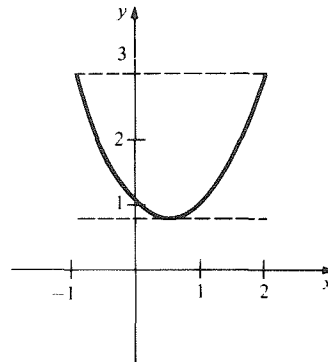


33. Maximize  $f(x) = x(M - x)$
35.  $l = 1$  is a minimum; the rectangle is a square of side length 1.
37.  $x = M/2$ ; the two masses should be equal.
39. 10 cm on each side.
41. (a)  $r = \sqrt[3]{500/\pi}$  cm,  $h = 2\sqrt[3]{500/\pi}$  cm.  
(b)  $r = \sqrt[3]{V/2\pi}$  cm,  $h = 2\sqrt[3]{V/2\pi}$  cm.  
(c)  $r = \sqrt{A/6\pi}$  cm,  $h = \sqrt{2A/3\pi}$  cm.
43. 749 units.
45. Height and width are 12 inches, length 24 inches.
47. 32, 4
49. (a) Circle of radius  $500/\pi$  feet.  
(b) Circle has radius  $500/(\pi + 4)$  feet, square has side  $1000/(\pi + 4)$ .
51. (a)  $y = \sqrt[3]{ab^2} + a$   
(b)  $(a^{2/3} + b^{2/3})^{3/2}$
53.  $l = (3 - \sqrt{6})c/3$
55. The square of side 1.
57. The semicircle with radius  $500/\pi$  meters.
59. The right triangle with legs of length  $\sqrt{2}$ .

61. One possibility is  $y = -|x|$ .
63. One possibility is  $f(x) = x$  on  $(-2, 2)$ ,  $f(2) = f(-2) = 0$ .
65.  $M_1 < M_2$  since  $M_1$  is a value and  $M_2$  is the maximum value.
67. For  $p > 0, q > 0$ ,  $(\sqrt{q/p}, 2\sqrt{p/q})$  is a minimum;  $p < 0, q < 0$ ,  $(\sqrt{q/p}, 2\sqrt{p/q})$  is a maximum. Otherwise, no solution.
69. There are none.
71.  $x \approx 0.497$  or  $1.503$ .
73. (a) A quart  
(b) If  $q \geq g/2$ ,  $G \leq Q + g/2$ ; if  $q \leq g/2$ ,  $G \leq Q + q$ .
75.  $f$  and  $g$  must have a maximum at the same point.
77. Use the closed interval test and the definition of concavity.

### 3.6 The Mean Value Theorem

1. Use the mean value theorem.
3.  $f(x) = 2g(x) + C$
5. Use the mean value theorem.
7. Use the mean value theorem.
9. It is between 72 and 76.
11.  $x_0 = 1/2$ .

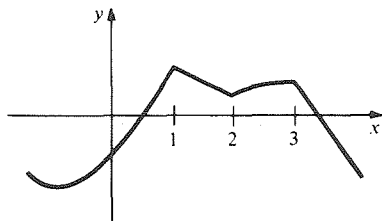


13.  $F(x) = (1/x) + C_1$  for  $x < 0$ ;  $F(x) = (1/x) + C_2$  for  $x > 0$ ;  $C_1$  doesn't necessarily equal  $C_2$
15. No,  $f'(x)$  doesn't exist at  $x = 0$ .
17.  $x^2/4 - 4x^3/3 + 21x + C$ .
19.  $-1/x + x^2 + C$ .
21.  $2x^5/5 + 8/5$
23.  $x^5/5 + x^4/4 + x^3/3 + 13/60$
25. Use the mean value theorem twice.
27. Use the horserace theorem.
29. Show that  $dN/dt \geq 0$  cannot hold on  $(t_1, t_2)$  by using the fact that  $N$  is nonconstant.

### Review Exercises for Chapter 3

1.  $(-\infty, -1)$  and  $(1, \infty)$
3.  $(-\infty, 1)$ ,  $(1, 2)$  and  $(2, \infty)$
5. Use the definition of continuous functions. Yes, polynomials are continuous and  $h(x)$  is continuous at  $x = 2$ .

7.



9. Use the intermediate value theorem.

11. The root is approximately 0.83.

 13. Increasing on  $(-\infty, 0)$  and  $(1/4, \infty)$ ; decreasing on  $(0, 1/4)$ .

 15. Increasing on  $(-\sqrt{2}, \sqrt{2})$ ; decreasing on  $(-\infty, -\sqrt{2})$  and  $(\sqrt{2}, \infty)$ .

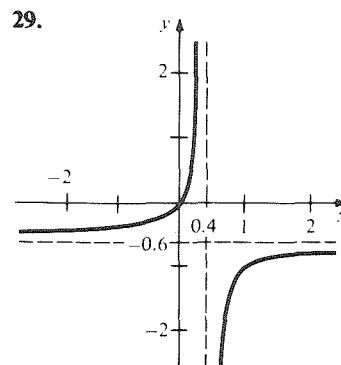
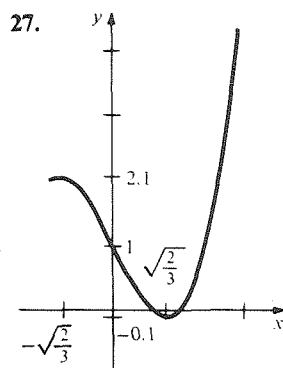
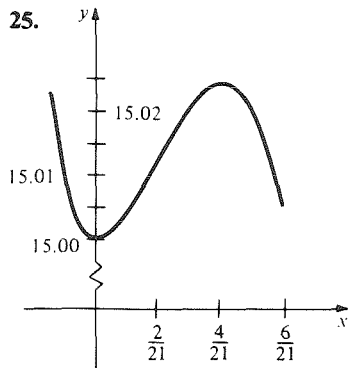
 17. Increasing for  $t < -5/3$  and  $t > 5$ , decreasing for  $-5/3 < t < 5$  for  $t$  in  $[-3, 7]$ . Political reaction to minimum: fulfilled promises; maximum: things are turning around now; inflection point: the rate at which things are getting worse has just turned around for the better.

 19. Speeding up on  $(50, 100)$ ; slowing down on  $(0, 50)$ .

 21.  $x_0 = 2/3$  is a local maximum;  $x_0 = 1$  is a local minimum.

 23.  $x_0 = -3$  and  $x_0 = 1$  are local minima;  $x_0 = -1$  is a local maximum.

|     | Continuous                          | Differentiable                      | Increasing                               | Decreasing                         | Concavity<br>Up Down               | Endpoints | Local<br>Maximum Minimum        | Inflection Points                           |
|-----|-------------------------------------|-------------------------------------|------------------------------------------|------------------------------------|------------------------------------|-----------|---------------------------------|---------------------------------------------|
| 25. | $(-\infty, \infty)$                 | $(-\infty, \infty)$                 | $(0, 4/21)$                              | $(-\infty, 0)$<br>$(4/21, \infty)$ | $(-\infty, 2/21)$ $(2/21, \infty)$ | —         | $x_0 = 4/21$<br>$x_0 = 0$       | $x_0 = 2/21$                                |
| 27. | $[-1, 2]$                           | $[-1, 2]$                           | $[-1, -\sqrt{2/3})$<br>$(\sqrt{2/3}, 2]$ | $(-\sqrt{2/3}, \sqrt{2/3})$        | $(0, 2]$ $[-1, 0)$                 | $-1, 2$   | $x_0 = \sqrt{2/3}$<br>$x_0 = 2$ | $x_0 = \sqrt{2/3}$<br>$x = -1$<br>$x_0 = 0$ |
| 29. | $(-\infty, 2/5)$<br>$(2/5, \infty)$ | $(-\infty, 2/5)$<br>$(2/5, \infty)$ | $(-\infty, 2/5)$<br>$(2/5, \infty)$      | —                                  | $(-\infty, 2/5)$ $(2/5, \infty)$   | —         | —                               | —                                           |



31. False.

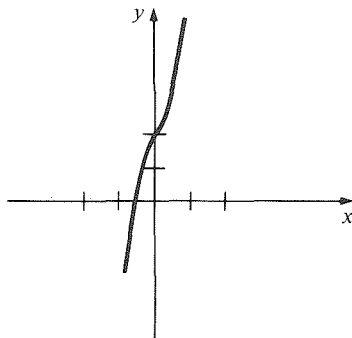
33. False.

35. False.

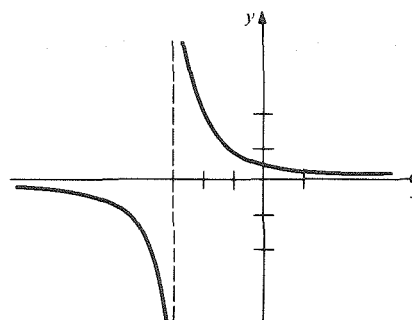
37. True.

39. True.

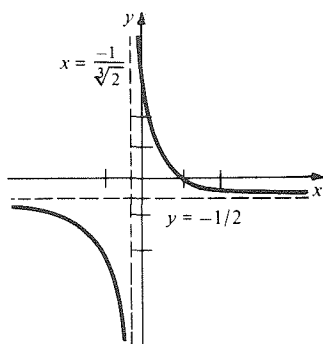
41.



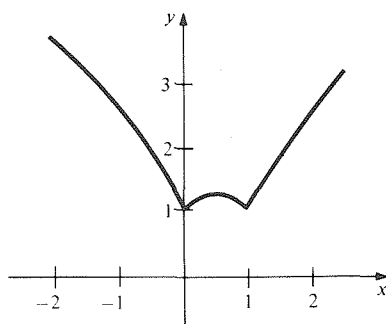
43.



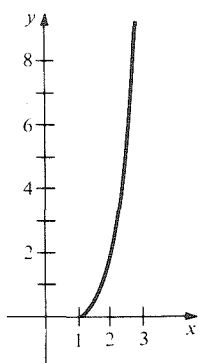
45.



47.



49.

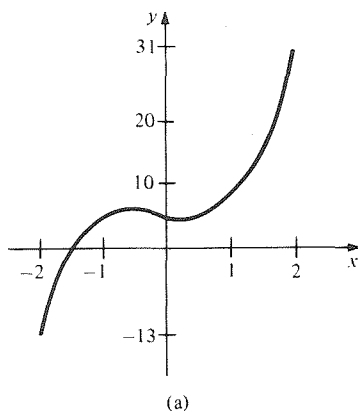


51. Maximum value is 31; minimum value is -13.

53. Maximum value is 3; minimum value is  $2\sqrt{30} - 10$ .

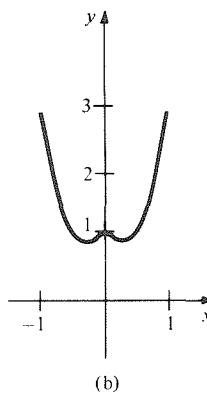
55. There are none.

57.



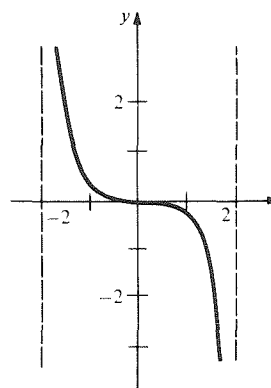
(a)

59.



(b)

61.


63.  $(5\sqrt{2} + 2)$  inches by  $(10\sqrt{2} + 4)$  inches.

65. Height is  $h/3$ , radius is  $2r/3$ .

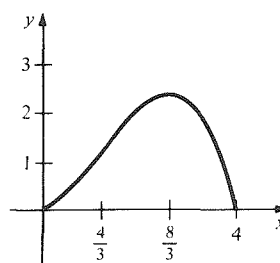
67.  $l = \sqrt[3]{5V/2}$ ,  $h = (2\sqrt[3]{5V/2})/5$ ; the ratio  $l/h$  is independent of  $V$ .

69.  $V = 2\pi r^3 + 40\pi r^2$  yields radius,  $h = V/\pi r^2$ .

71.  $x = \sqrt{1/4 + (\pi/10)^2}$ ,  $P = 180/(5 + \sqrt{\pi^2 + 25})$ 

73. (a) 1

(b)


75. (a) Show that the minimum occurs at  $(a + b)/2$  and use the fact  $[(a + b)/2]^2 \geq ab$  for positive  $a$  and  $b$ .

77. (a)  $R$  = price per item  $\times$  number of items.  $C$  = initial cost + (cost per item)  $\times$  (number of items).  $P$  = cash received - cost.

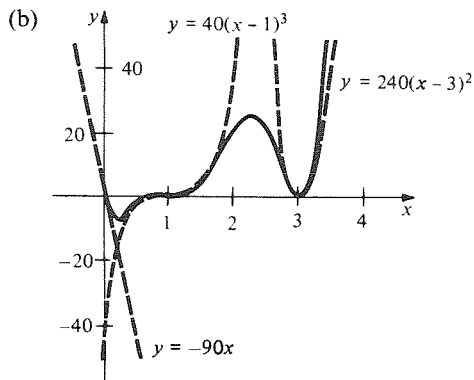
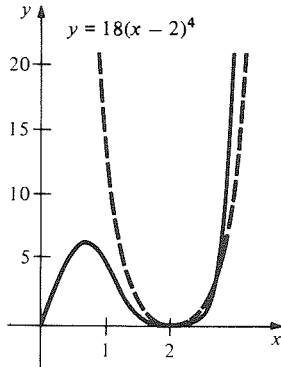
(b)  $x = 2000$  units.

(c)  $x = 3000$  units.

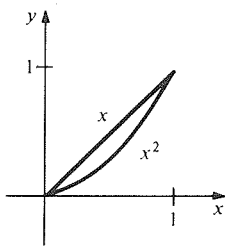
(d)  $x = 5,625,000$  units.

79.  $l = 16\sqrt{5}$  feet,  $w = 10\sqrt{5}$  feet.

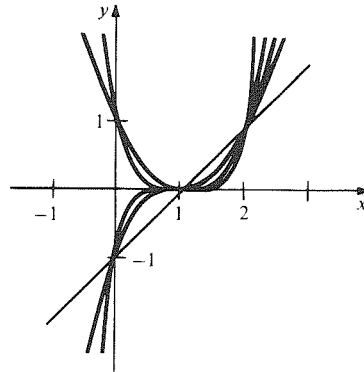
81. (a)  $x = 40/(\sqrt[3]{3} + 1)$   
 (b) 16.4 miles
83. If  $f'''(x) = 0$ , then  $f''(x) = 2A$ , a constant. Then  $f'(x) - 2Ax$  is a constant too.
85. (a) 119.164 lbs.  
 (b) Gain is 7.204 lbs.
87. (a) Set the first derivative equal to 0.  
 (b)  $fg$  has a critical point if  $f'/f = -g'/g$ .
89. (a)



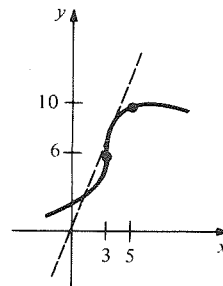
91. (a)



- (b) Use the definitions.  
 (c) Use properties of exponentiation.  
 (d) Use properties of exponentiation.  
 (e) Consider the cases  $|x - a| < 1$  or  $|x - a| \geq 1$  and when  $m, n$  are  $< 1$  or  $\geq 1$ .  
 (f)



93. Assume that there is no maximum or minimum and show that this contradicts  $f(0) = f(1)$ .
95. The maximum and minimum points of  $f$  on  $[a, b]$  cannot both be endpoints.
97. Use the mean value theorem on  $[f(x)]/x$ .

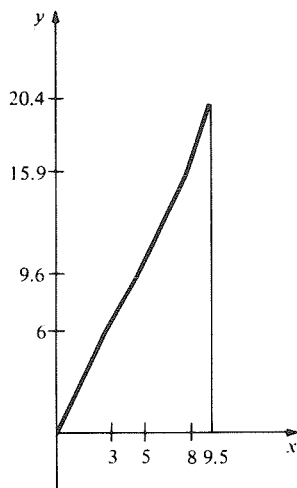


99. Consider the second derivative of  $h(x) = x^2 f(x)$ .

# Chapter 4 Answers

## 4.1 Summation

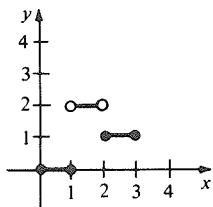
1. 20.4 meters.
3. 91.0 meters.
5. 34
7. 40
9. 325
11. 1035
13. 3003
15. 9999
17. 0
19. 5865
21.  $[n(n+1) - (m-1)m]/2$
23. Notice that  $1/(1+k^2) \leq 1$  for  $k \geq 1$ .
25. 100,000,000
27. 10,400
- 29.



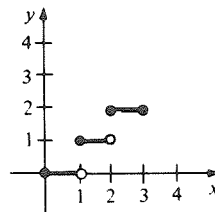
31. 14,948
33.  $(n+1)^4 - 1$
35. 30,600
37. 122
39. 124
41. (a) Apply the telescoping sum formula.  
(b)  $[n(n+1)(2n+1)]/6 - [(m-1)(2m-1)m]/6$   
(c)  $[n(n+1)/2]^2$

## 4.2 Sums and Areas

1.

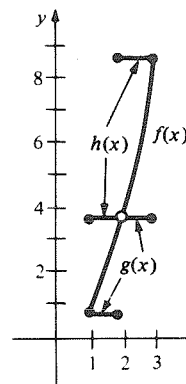


3.



|    | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $\Delta x_1$ | $\Delta x_2$ | $\Delta x_3$ | $k_1$ | $k_2$ | $k_3$ | Area |
|----|-------|-------|-------|-------|--------------|--------------|--------------|-------|-------|-------|------|
| 5. | 0     | 1     | 2     | 3     | 1            | 1            | 1            | 0     | 2     | 1     | 3    |
| 7. | 0     | 1     | 2     | 3     | 1            | 1            | 1            | 0     | 1     | 2     | 3    |

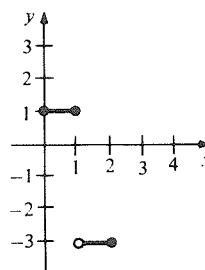
9.  $L = 5$ ,  $U = 13$



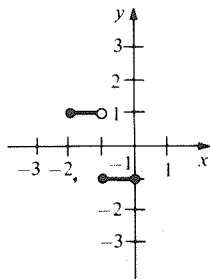
11.  $91.5 \text{ meters} \leq d \leq 117 \text{ meters}$ .
13.  $0.01 \text{ meters} \leq d \leq 0.026 \text{ meters}$ .
15.  $3/2$
17.  $5(b^2 - a^2)/2$
19.  $1/2$
21.  $11/6$
23.  $1/6$

## 4.3 The Definition of the Integral

1. -2



3. 0



5. -2

7. 0

9.  $\int_{-1}^2 x^2 dx$

11.  $\int_{-1}^2 x dx$

13.  $7/10$

15. 0.509

17. (a)  $\int_0^3 5(t^2 - 5t + 6) dt$

(b)  $\int_0^2 5(t^2 - 5t + 6) dt - \int_2^3 5(t^2 - 5t + 6) dt$

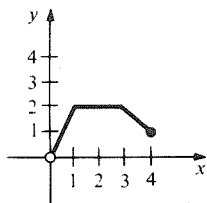
19.  $\lim_{n \rightarrow \infty} (1/n^6) \sum_{i=1}^n i^5$

21.  $\lim_{n \rightarrow \infty} \left( \sum_{i=1}^n \frac{2n}{5n^2 + 8in + 4i^2} \right)$

23. Choose step functions with two steps.

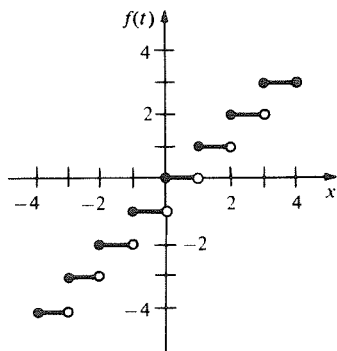
25. (a)  $\int_0^x f(t) dt = \begin{cases} 2x & \text{if } 0 \leq x < 1 \\ 2 & \text{if } 1 \leq x < 3 \\ -x + 5 & \text{if } 3 \leq x \leq 4 \end{cases}$

(b)

(c)  $F$  is differentiable on  $(0, 4]$  except at 1, 3 and 4.

$$F'(x) = \begin{cases} 2 & \text{if } 0 < x < 1 \\ 0 & \text{if } 1 < x < 3 \\ -1 & \text{if } 3 < x < 4 \end{cases}$$

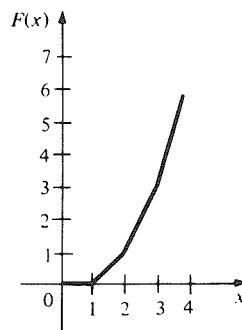
27. (a)



(b) 0, 15, -2, 8

(c)  $(n-1)n/2$ 

(d)



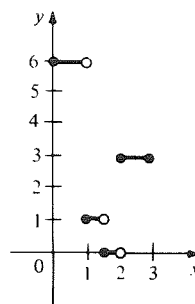
$$F'(x) = \begin{cases} 0 & 0 < x < 1 \\ 1 & 1 < x < 2 \\ 2 & 2 < x < 3 \\ 3 & 3 < x < 4 \end{cases}$$

29. (a)  $\sum_{i=1}^n A_i \Delta x_i$

(b)  $\int_0^L f(x) dx$

31. (a) Choose a partition and consider  $kf(x)$  on each interval.

(b)  $k \int_a^b f(x) dx$

33. (a)  $19/2$ (b)  $1/2$ 

(c) They are both 19.

(d) Calculate each explicitly.

(e) No.

35.  $1/6$ 

## 4.4 The Fundamental Theorem of Calculus

1. 20

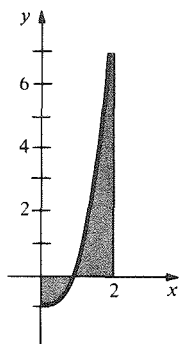
5.  $8^{1/4}$ 9.  $(3/7)(b^{7/3} - a^{7/3})$ 13.  $-400/3$ 17.  $11/1800$ 21.  $319/24$ 25.  $7/3$ 

3. 30

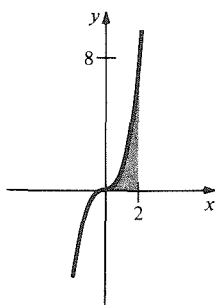
7. 24

11.  $(12\pi/5)(\sqrt[3]{32} - 1)$ 15.  $78/5$ 19.  $29/6$ 23.  $59/6$ 27.  $3/4$

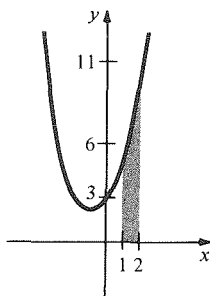
29. 2



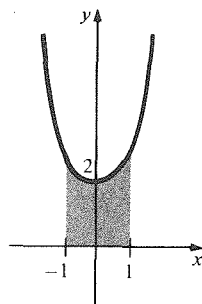
31. 4



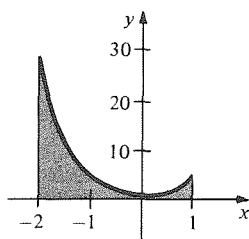
33. 25/3



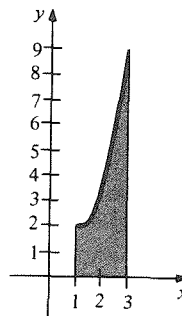
35. 22/5



37. 93/5



39. 82/9



41. 604,989/5 units.

43.  $-29/3$  units, 16 units.

45. 1600 feet.

47. (a)  $v_0^2/g$   
(b)  $5v_0^2/8g$ 

49. On an interval  $(x_i, x_{i+1})$  on which  $h(t) = l_i$ ,  $f(t) \leq l_i$ , so  $F(x_{i+1}) - F(x_i) \leq (\Delta x_i)l_i$  by the mean value theorem. Now use a telescoping sum.

## 4.5 Definite and Indefinite Integrals

1.  $(d/dx)(x^5) = 5x^4$

3.  $(d/dx)(t^{10}/2 + t^5) = 5t^9 + 5t^4$

5. (a)  $(d/dx)[t^3/(1+t^3)] = 3t^2/(1+t^3)^2$

(b)  $1/2$

7. (a)  $(3x^2 + x^4)/(x^2 + 1)^2$

(b)  $1/2$

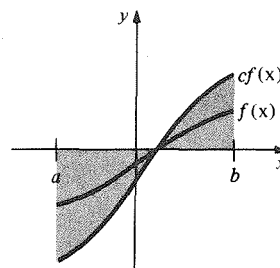
9. 370/3

11. 16512/7

13. 11/6

15. 1/6

17. 3724/3

19. (a) For any  $c$ , the area under  $c$  times  $f(x)$  is  $c$  times the area under  $f(x)$ .

(b) If you go  $c$  times as fast, you go  $c$  times as far.

21. 7

23. -8

25.  $-5/2$

27.  $-199/16200$

29.  $\frac{d}{dx} \left( \frac{x^4}{4} - x - \frac{a^4}{4} + a \right) = x^3 - 1$

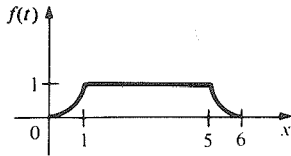
31. 1/512

33.  $3/(t^4 + t^3 + 1)^6$

35.  $-t^2(1+t)^5$

37. Differentiating the distance function with respect to time gives the velocity function.

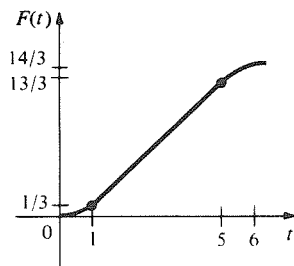
39. (a)



(b)  $14/3$

(c)  $14/3$

$$(d) F(t) = \begin{cases} t^3/3 & 0 \leq t \leq 1 \\ t - 2/3 & 1 \leq t \leq 5 \\ (t-6)^3/3 + 14/3 & 5 \leq t \leq 6 \end{cases}$$



$$(e) F'(t) = \begin{cases} t^2 & 0 < t \leq 1 \\ 1 & 1 \leq t \leq 5 \\ (t-6)^2 & 5 \leq t < 6 \end{cases}$$

41. (a)  $\frac{d}{dt} [F_1(t) - F_2(t)] = f(t) - f(t) = 0$

(b)  $\int_{a_1}^{a_2} f(s) ds$

43. (a) Let  $u = g(t)$  and  $G(u) = \int_a^u f(s) ds$ . Then  $\int_a^{g(t)} f(s) ds = G(g(t))$ . Now use the chain rule.

(b) The rate of change of area = height times the speed of the screen.

45.  $2/x$

47.  $f(g(t)) \cdot g'(t) - f(h(t)) \cdot h'(t)$ . The rate of change of area as both endpoints move is the sum of the rates due to the motion of each endpoint.

49. The general fact about inequalities is: if  $c < a + b$  then  $c = c_1 + c_2$  where  $c_1 < a$  and  $c_2 < b$ . (Let  $c_1 = a - \frac{1}{2}(a + b - c)$  and  $c_2 = c - c_1$ .)

51.  $\int_a^{t+h} f(s) ds - \int_a^t f(s) ds = \int_t^{t+h} f(s) ds$

53.  $f(c) \rightarrow f(t)$  as  $h \rightarrow 0$  by continuity of  $f$ .

## 4.6 Applications of the Integral

1.  $160/3$

5.  $16/3$

9.  $1/6$

13.  $141/80$

17.  $207/4$

21.  $32/3$

25. 12,500 liters.

29. (a)  $bh/2$

(b)  $bh/2$

31. Speedometer.

33. 15 minutes, 43 seconds.

35. (a) Integrate both sides of

$$W'(t) = 4(t/100) - 3(t/100)^2.$$

(b)  $t^2/50 - t^3/10,000$

(c) 100 words

37.  $y = (11 - 4\sqrt{6})/8$

3. \$18,856

7.  $31/5$

11.  $3/10$

15. 8

19.  $1/4, 1/4$

23.  $1/8$

27. 7200

## Review Exercises for Chapter 4

1. 30

5. 379,250

9.  $223/60$

3.  $21/5$

7. 14,641

11. If  $x$  is time in seconds,  $\int_0^1 |f(x)| dx$  is the distance travelled where  $f(x)$  is the velocity.

13.  $-718/3$

15.  $-5/4$

17.  $7/12$

19.  $(3/7)(5^{7/3} - 3^{7/3})$

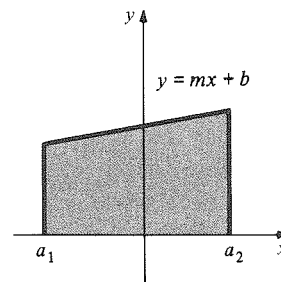
21. (a) 3.2399, 3.0399.

(b) 3.1399. [The exact integral is  $\pi$ .]

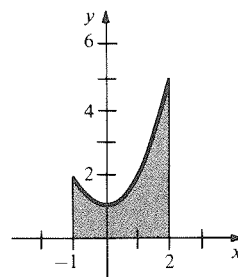
23. (a)  $-2x/(1+x^2)^2$

(b)  $1/4$

25.  $(a_2 - a_1)(ma_1 + b + ma_2 + b)/2$



27. 6



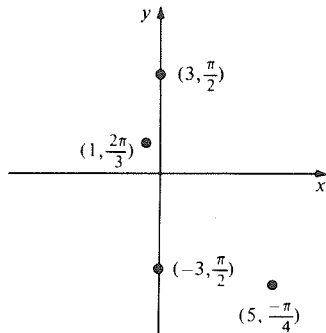
## A.30 Chapter 4 Answers

29. (a) Differentiate the right-hand side.  
(b)  $2/63$
31.  $64/21$
33.  $125/6$
35. The plane was at least 1700 feet above ground.
37. (a) 22 liters.  
(b) 16 liters.  
(c)  $(256/27)$  liters.
39. (a)  $\int_0^a [D(x) - b] dx$  is the area between the curves  $p = D(x)$  and  $p = b$ , from  $x = 0$  to  $x = a$ .  
(b)  $\int_0^a [b - S(x)] dx$  is the area between the curves  $p = b$  and  $p = S(x)$  from  $x = 0$  to  $x = a$ .  
(c) Interpret the integral in (a) in terms of sums.  
(d) Consumer's surplus =  $\frac{1}{2}$ , Producer's surplus =  $\frac{2}{3}$ .
41.  $110/3$
43. (a)  $\int_0^t 32x dx = 16t^2$  feet;  $\int_0^t 1080 dx = 1080t$  feet.  
(b) 5.2 seconds.  
(c) 432.56 feet.  
(d) 0.4 seconds.
45. (a) The fifth day.  
(b)  $-4000$  bacteria per cubic centimeter.
47. 4
49.  $x^2/(1 + x^3)$
51.  $15/2$
53.  $3t^2/[(t^3 + 2)^2 + 1]$
55. (a) Since  $f$  is unbounded at 0, it has no upper sum and is not integrable.  
(b)  $\int_\epsilon^1 t^{-1/2} dt + \delta$  is a lower sum for  $\epsilon > 0$  and  $\delta > 0$ .  
(c)  $\epsilon f(\epsilon) + \int_\epsilon^1 f(t) dt = 2 - \sqrt{\epsilon}$  is a lower sum for  $\epsilon > 0$ .  
(d) 2

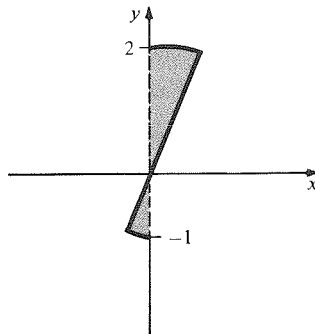
# Chapter 5 Answers

## 5.1 Polar Coordinates and Trigonometry

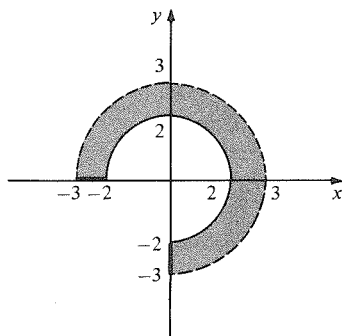
1.  $3.84 \text{ m}$ ,  $19.19 \text{ m}^2$   
 3.  $5/18 \text{ rad}$ ,  $45 \text{ m}^2$   
 5.  $0.5061$ ,  $0.9425$ ,  $4.4506$ ,  $2.2689$ ,  $5.5851$ .  
 7. (a)  $\pi/3$ ,  $6\pi/5$ ,  $\pi$ .  
 (b)  $160^\circ$ ,  $305^\circ$ ,  $25^\circ$   
 9.



11.



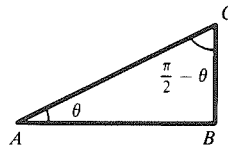
13.

15.  $(5.3852, -0.3805)$ 

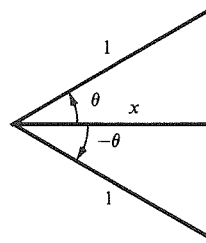
17. (a)  $(1, 0)$  (b)  $(5, 0.927)$   
 (c)  $(2, \pi/6)$  (d)  $(2, -\pi/6)$   
 (e)  $(2, 5\pi/6)$

19. (a)  $(\sqrt{2}, -\pi/4)$  (b)  $(2, \pi/2)$   
 (c)  $(\sqrt{197}/2, 1.5)$  (d)  $(13, -2.75)$   
 (e)  $(\sqrt{73}, 1.93)$  (f)  $(3\sqrt{2}/4, \pi/4)$

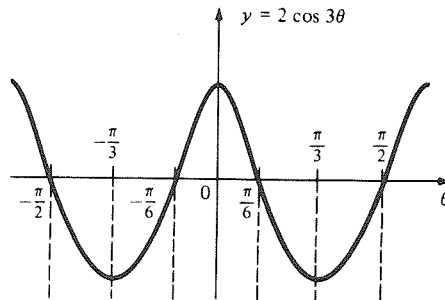
21. (a)  $(0, 6)$  (b)  $(6\sqrt{2}, -6\sqrt{2})$   
 (c)  $(-4, 0)$  (d)  $(0, 2)$   
 (e)  $(-4, -4\sqrt{3})$  (f)  $(0.42, -0.91)$   
 23.  $\tan \theta = |BC|/|AC| = \cot(\pi/2 - \theta)$



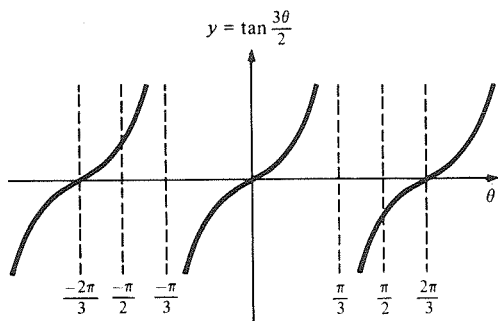
25. In the Figure,  $\cos \theta = x/1 = \cos(-\theta)$ .



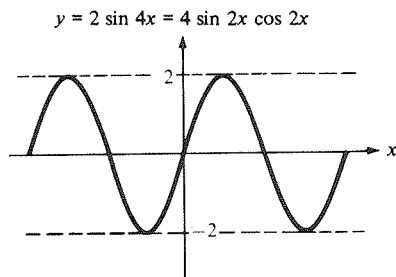
27.  $84.5$   
 29.  $8\sqrt{2}$   
 31.  $10,723 \text{ feet}$ .  
 33.  $917.19 \text{ meters}$ .  
 35.  $1.313$   
 37.  $.3 \text{ radians}$   
 39. Write  $\cos \theta = \cos\left(\frac{\theta}{2} + \frac{\theta}{2}\right)$  and expand.  
 41. Use the addition formulas on the right-hand side.  
 43.  $\cos \theta$   
 45.  $-\sin \theta$   
 47.  $\sec \theta$   
 49.  $-\sin \theta \cos \phi$   
 51.  $(2 + \sqrt{2 + \sqrt{3}})^{1/2}/2$   
 53.  $2/\sqrt{2 + \sqrt{3}}$   
 55. Use  $\cos[2(\theta/2)] = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$ .  
 57. Take the reciprocal of both sides and use  $\cos^2(\theta/2) = (1 + \cos \theta)/2$ .  
 59. Take the reciprocal of both sides and use the product formula for  $\sin \theta \sin \phi$ .  
 61.



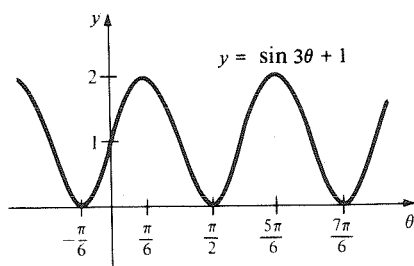
63.



65.



67.


69.  $(2n + 1)\pi/2$  where  $n$  is an integer.

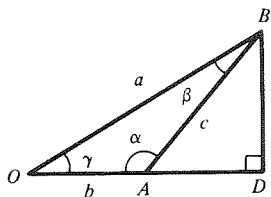
71.  $\sec \theta$  is differentiable for all  $\theta \neq (2n + 1)\pi/2$ ,  $\cot \theta$  is differentiable for all  $\theta \neq n\pi$ ,  $n$  an integer.

73. (a)  $\sqrt{3} \times 10^{10}$  meters per second.

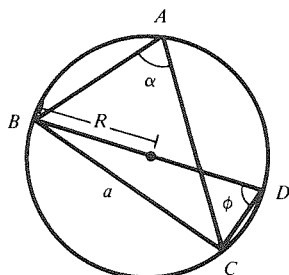
(b) The angle of incidence equals the angle of refraction.

(c)  $20.7^\circ$  or  $0.36$  radians.

75. Consider this figure:



77. Consider this figure:

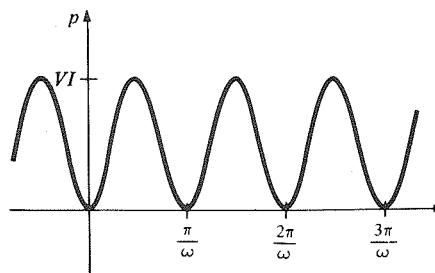


79. Expand the right-hand side using the addition formula.

81. (b)  $\sin \theta = n\lambda/a$ .

83. (a) Use  $\sin^2(2\omega t/2) = [1 - \cos(2\omega t)]/2$ 

(b)



## 5.2 Differentiation of the Trigonometric Functions

1.  $-\sin \theta + \cos \theta$ 

3.  $-15 \sin 3\theta + 20 \cos 2\theta$ 

5.  $\cos 2\theta - \theta \sin \theta + \cos \theta$ 

7.  $-9 \cos^2 3\theta \sin 3\theta$ 

9.  $\sin \theta / (\cos \theta - 1)^2$ 

11.  $(\cos \theta - \sin \theta - 1) / (\sin \theta + 1)^2$ 

13.  $-3 \cos^2 x \sin x$ 

15.  $4(\sqrt{x} + \cos x)^3 [(1/2\sqrt{x}) - \sin x]$ 

17.  $(1 + 1/2\sqrt{x}) \cos(x + \sqrt{x})$ 

19.  $\frac{\cos x + \sin(x^2) + x \sin x - 2x^2 \cos(x^2)}{[\cos x + \sin(x^2)]^2}$ 

21.  $\sec^2 x - 2 \sin x$ 

23.  $3 \tan 3x \sec 3x$ 

25.  $1/2\sqrt{x} - 3 \sin 3x$ 

27.  $-\sin x / 2\sqrt{\cos x}$ 

29.  $12t^2 \sin \sqrt{t} + [(4t^3 + 1)/2\sqrt{t}] \cos \sqrt{t}$ 

31.

$$-3x^2(\cos \sqrt{1-x^3}) + \frac{(-3x^4 + 1) \cdot \sec^2[x/(x^4 + 1)]}{(x^4 + 1)^2}$$

33.

$$\frac{-3}{2(\theta^2 + 1)^{3/2}} \sqrt{\csc\left(\frac{\theta}{\sqrt{\theta^2 + 1}}\right) + 1} \cot\left(\frac{\theta}{\sqrt{\theta^2 + 1}}\right) \csc\left(\frac{\theta}{\sqrt{\theta^2 + 1}}\right)$$

35.

$$v \sec\left(\frac{1}{v^2 + 1}\right) \left[ \frac{\sec^2 \sqrt{v^2 + 1}}{\sqrt{v^2 + 1}} - \frac{2 \tan \sqrt{v^2 + 1}}{(v^2 + 1)^2} \tan\left(\frac{1}{v^2 + 1}\right) \right]$$

37.  $x^4/4 - \cos x + C$ 

39.  $x^5/5 + (\sec 2x)/2 + C$ 

41.  $-2 \cos(u/2) + C$ 

43.  $[\sin(\theta^2)]/2 + C$ 

45.  $[-\cos(2\theta)]/4 + C$ 

47.  $\theta + C$ 

49.  $4 - 4 \cos(\pi/8)$ 

51. 0

53. 0

55.  $\pi/2$

57. (a) Prove  $\sin 2\phi/(1 + \cos 2\phi) = \frac{\sin \phi}{\phi} \cdot \frac{\phi}{\cos \phi}$

(b) Manipulate  $\cos \phi < (\sin \phi)/\phi < 1/\phi$ .

59. a

61. Differentiate twice and substitute.

63. Differentiate once and substitute.

65. Differentiate  $[-f(\cos \theta)]$  using the chain rule.

69. (a)  $-\sin x \cdot \phi(3x) + [3 \cos x / \cos(3x)]$

(b)  $\phi(1) - \phi(0)$

(c)  $4 - 4\phi(2x)\sin(2x) + 4/\cos^2(2x)$

71. (a)  $\sqrt{(dx)^2 + (dy)^2} = dt$  since speed  $\approx 1$

(b) Differentiate

(c) Multiply the equation in (b) by  $\sin \theta \sin' \theta - \cos \theta \cos' \theta$  and manipulate.

(d) If  $\cos \theta$  and  $\sin \theta$  are positive,  $\cos' \theta$  is negative and  $\sin' \theta$  is positive.

### 5.3 Inverse Functions

1.  $(x - 5)/2$  on  $[-3, 13]$

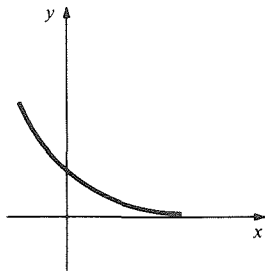
3.  $\sqrt[5]{x}$  on  $(-\infty, \infty)$

5.  $(3t + 10)/(1 - t)$  on  $[-11/2, -9/4]$

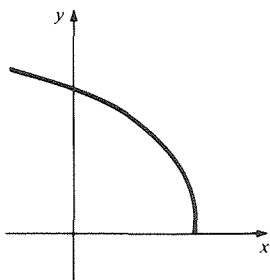
7.  $(-dx + b)/(cx - a)$ , domain  $x \neq a/c$ . The condition is  $bc - ad \neq 0$ .

9.

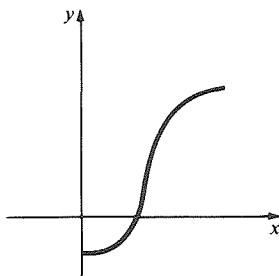
(a)



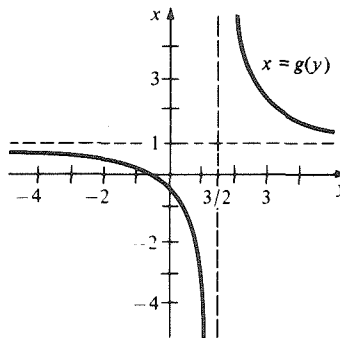
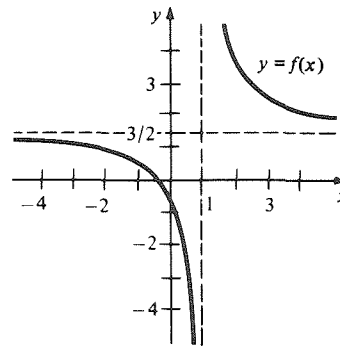
(b)



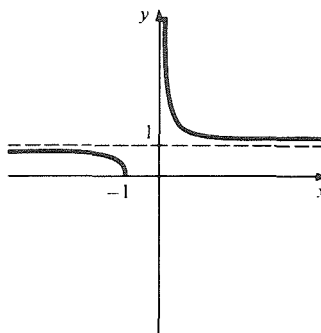
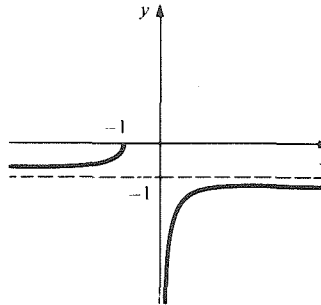
(c)



11.



13. Largest interval is  $(-\infty, -1)$  or  $(1, \infty)$ .



15. (a)  $[0, 8/3]$

(b)  $2, -1/4$

(c)  $[-229/27, 1]$

(d)  $1.572, -0.194$

17. Show that 1 is a local minimum point for  $f$ .

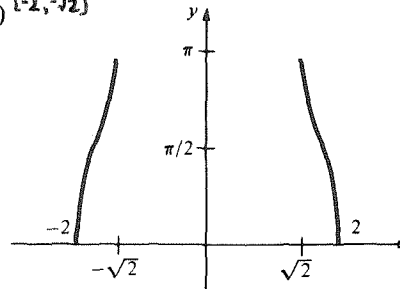
## A.34 Chapter 5 Answers

19. Show that  $f$  is increasing everywhere. The domain is  $(-\infty, \infty)$ .
21. Show that  $f$  is decreasing on  $[-1, 2]$ . The domain is  $[-11, 4]$ .
23. Differentiate  $x^{1/3}$ .
25. Show that  $f$  is increasing on  $(0, 2)$ .  $(f^{-1})'(4) = \frac{1}{5}$ .
27.  $1/3$
29.  $1/3, 1/3$
31.  $-1, -4/3$
33. Different calculators may respond differently.
35. The inverse function gives the cost of  $y$  pounds of beans.
37. (a)  $(-\infty, -5/2], (-7/3, +\infty)$   
(b)  $(5 - 7y^2)/(3y^2 - 2)$   
(c)  $-1/[2(3x + 7)^{3/2}\sqrt{2x + 5}]$
39. Use the definitions.
41. (a)  $\Delta y/\Delta x$  is close to  $f'(x_0)$  if  $\Delta x$  is small.  
(b) Manipulate the inequality in (a).  
(c) Use the definitions.

## 5.4 The Inverse Trigonometric Functions

1. 0.615
3. -0.903
5. 0
7.  $\pi/3$
9.  $\pi/6$
11.  $8/\sqrt{1 - 64x^2}$
13.  $2x \sin^{-1}x + x^2/\sqrt{1 - x^2}$
15.  $2(\sin^{-1}x)/\sqrt{1 - x^2}$
17. 
$$\frac{(1 - x^2)(10x^4 + 1) + 2x^2(2x^4 + 1)}{(1 - x^2)^2 + x^2(2x^4 + 1)^2}$$
19.  $[1 + (2/y^3)]/\sqrt{[y - (1/y)]^2[(y - (1/y^2))^2 - 1]}$
21.  $[3(x^2 + 2) - (2x)(\sin^{-1}3x)\sqrt{1 - 9x^2}]/[(x^2 + 2)^2 \cdot \sqrt{1 - 9x^2}]$
23. 
$$\frac{2t^{11} - t^8 + 12t^7 + t^4 - 2}{(t^5 + 2t)\sqrt{(t^5 + 2t)^2 - (t^7 + t^4 + 1)^2}}$$
25.  $(\sin^{-1}x + \cos^{-1}x - 1)/(1 - \sin^{-1}x)^2\sqrt{1 - x^2}$
27.  $\frac{3}{2}(x^2\cos^{-1}x + \tan x)^{1/2}$   
 $\times [2x\cos^{-1}x - \frac{x^2}{\sqrt{1 - x^2}} + \sec^2x]$
29.  $3\tan^{-1}x + x^2/2 + C$
31.  $4\sin^{-1}x + C$
33.  $3\tan^{-1}(2x)/2 + C$
35.  $2\sec^{-1}y + C$
37. Draw a right triangle with sides 1,  $x$ ,  $\sqrt{1 - x^2}$ .
39. No.
41.  $-\theta/(1 - \theta^2)^{3/2}$

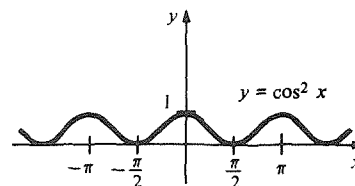
43.  $4\sin^{-1}(2x)/\sqrt{1 - 4x^2} + 2x$
45. 0
47. Minimum at  $x = -1$ , maximum at  $x = 1$ , point of inflection at  $x = 0$ .
49. Use  $(d/dy)[f^{-1}(y)] = 1/[(d/dx)f(x)]$ .
51. (a)  $[\sqrt{2}, 2], f'(x) = -2x/\sqrt{-x^4 + 6x^2 - 8}$   
(b)  $[-2, -\sqrt{2}]$



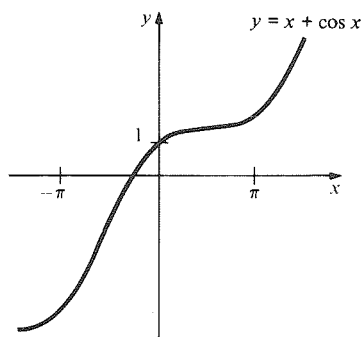
53. (a)  $(\sqrt{1 - x^2y^2} - y)/(x - \sqrt{1 - x^2y^2})$   
(b)  $\frac{[\cos(x + y) - y][1 + t^2]}{[x - \cos(x + y)][1 - t^2]^2}$   
(c)  $[x - \cos(x + y)]/[\cos(x + y) - y]\sqrt{1 - t^2}$   
(d)  $[\cos(x + y) - y](3t + 2)/[x - \cos(x + y)]$
55.  $\tan^{-1}(2) - \pi/4$
57.  $(3 - \sqrt{2})/2 + \pi/12 + \sin(\sqrt{2}/2) - \sin(1/2)$
59.  $-1/\sqrt{1 - y^2}$

## 5.5 Graphing and Word Problems

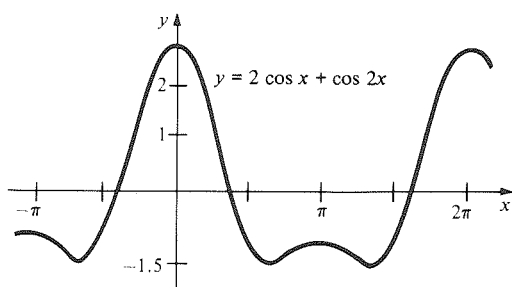
1. -0.088 radians/second.
3.  $(2.5)\pi$  meters/second.
5. 2.2 meters/second.
7. 59.2 meters before the sign.
9. 6
11. (a)  $r = 1.5, \omega = 2\pi/3.1$   
(b)  $3\pi/3.1$
13.  $3\sin\theta = \sin\phi$
15. Maxima at  $x = (2n + 1)\pi/4$  for  $n$  an even integer.  
Minima at  $x = (2n + 1)\pi/4$  for  $n$  an odd integer.  
Points of inflection are at  $x = n\pi/2, n$  an integer.  
Concave up on  $(n\pi/2, (n + 1)\pi/2), n$  an odd integer.  
Concave down on  $(n\pi/2, (n + 1)\pi/2), n$  an even integer.
17.  $f(x)$  concave down in  $(2n\pi, (2n + 1)\pi)$  for  $x > 0$  and in  $(-(2n + 1)\pi, -2n\pi)$  for  $x < 0, n$  an integer.  
 $f(x)$  is concave up everywhere else.
- 19.



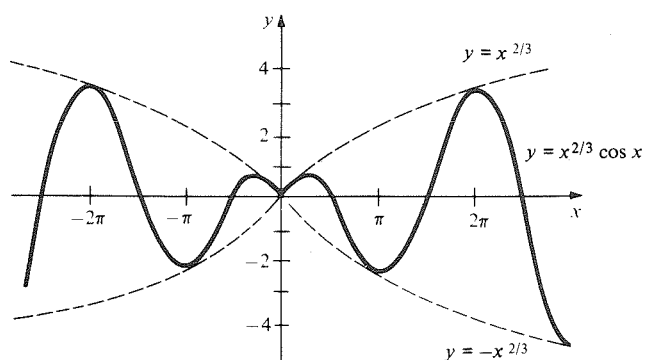
21.



23.



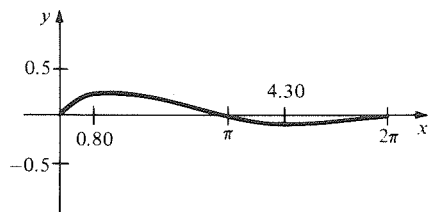
25.



27. The sum of the distances from  $(0, 1)$  to  $(x, 0)$  and  $(p, -q)$  to  $(x, 0)$  is minimized when these points lie on a straight line.

29.  $y = -3x + \pi$ ,  $y = x/3 + \pi$

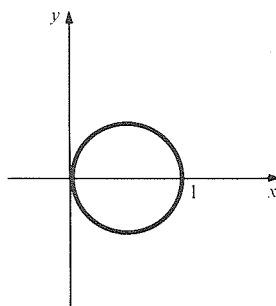
31.



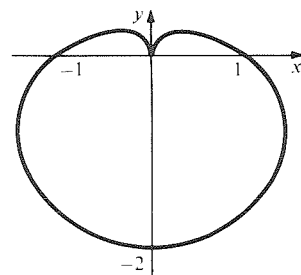
33. The local maxima and minima are about  $\pi$  units apart as  $x \rightarrow \infty$ .

## 5.6 Graphing in Polar Coordinates

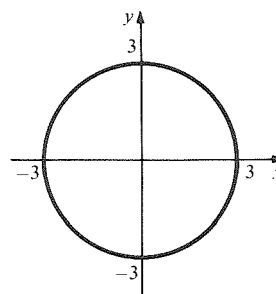
1.  $(x - \frac{1}{2})^2 + y^2 = \frac{1}{4}$



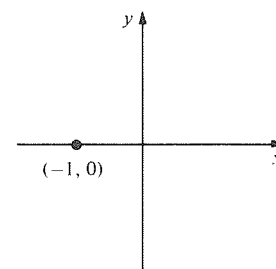
3.  $x^2 + y^2 = \sqrt{x^2 + y^2} - y$



5.  $x^2 + y^2 = 9$

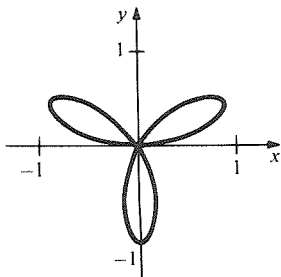


7.  $x = -1, y = 0$

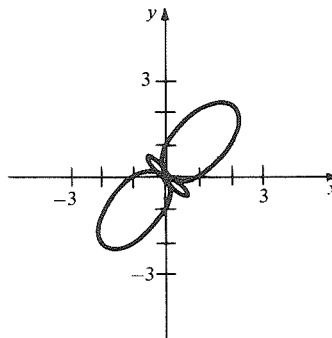


**A.36 Chapter 5 Answers**

9.  $(x^2 + y^2)^2 = 3x^2y - y^3$



37. max = 3, min = -1



11. A circle of radius  $r$  centered at the origin.

13. The graph is symmetric with respect to the line  $\theta = \pi/4$ .

15.  $r = 1$

17.  $r^2(1 + \cos \theta \sin \theta) = 1$

19.  $r \sin \theta (1 - r^2 \cos^2 \theta) = 1$

21.  $r(\sin \theta - \cos \theta) = 1$

23.  $-5\sqrt{3}$

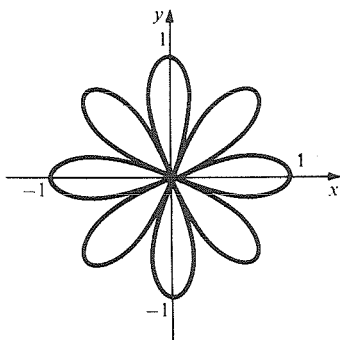
25. 0

27.  $13\sqrt{3}/9$

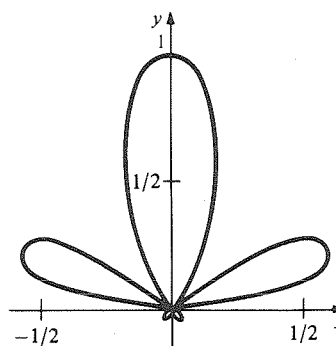
29.  $1/3$

31. -0.08

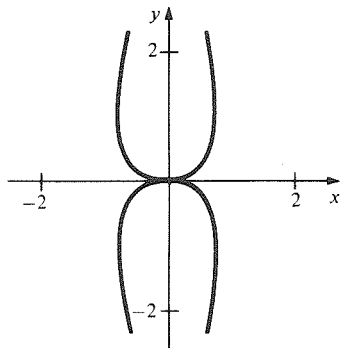
33. max = 1, min = -1



39. max = 1, min = -1



35. max =  $\infty$ , min =  $-\infty$



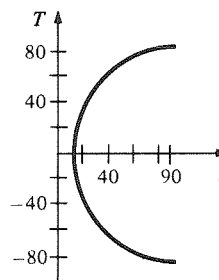
### Supplement to Chapter 5

1. 8:05:30 on July 13 and 8:05 on July 14.

3.  $\frac{d^2S}{dT^2} = \frac{-48\pi}{(365)^2} \tan l \sin \alpha \cos\left(\frac{2\pi T}{365}\right) \times$

$$\frac{(1 - a \cos^2 x)(1 - b \cos^2 x) - b \sin^2 x(1 - a \cos^2 x) - 2a \sin^2 x(1 - b \cos^2 x)}{\left[1 - \sin^2 \alpha \cos^2\left(\frac{2\pi T}{365}\right)\right]^2 \left[1 - \sin^2 \alpha \sec^2 l \cos^2\left(\frac{2\pi T}{365}\right)\right]^{3/2}}$$

5.  $T = \frac{365}{2\pi} \cos^{-1} \sqrt{\frac{0.107}{\tan^2 l + 0.017}}$



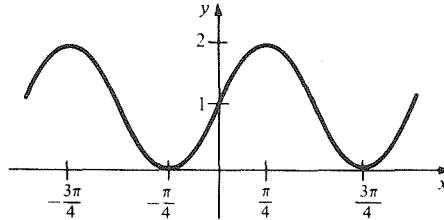
7. 5:07:47 P.M.  
 9. (a) 0.24 rad/hr.  
 (b) 21 minutes, 49 seconds.

## Review Exercises for Chapter 5

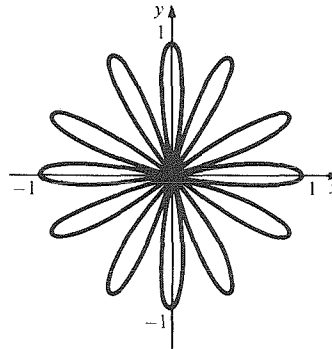
1. 1.152 radians.  
 3. (0, 4)  
 5.  $\tan \theta \sec \theta = r$   
 7.  $\tan(\theta + \varphi) = \frac{\sin(\theta + \varphi)}{\cos(\theta + \varphi)}$ ; expand and manipulate.  
 9. 1.392  
 11. 2.250  
 13. 0.38 radians.  
 15.  $-6 \cos 2x$   
 17.  $1 + \sin 3x + 3x \cos 3x$   
 19.  $2\theta + \csc \theta - \theta \cot \theta \csc \theta$   
 21.  $3y^2 + 2 \tan(y^3) + 6y^3 \sec^2(y^3)$   
 23.  $[-\sin(x^8 - 7x^4 - 10)](8x^7 - 28x^3)$   
 25.  $2(1 + \cos x)/(x + \sin x) \sqrt{(x + \sin x)^4 - 1}$   
 27.  $-36\theta \cos^2(\theta^2 + 1) \sin(\theta^2 + 1)$   
 29.  $1/2\sqrt{x - x^2}$   
 31.  $\cos \sqrt{x} \sec^2(\sin \sqrt{x})/2\sqrt{x}$   
 33.  $(1/2\sqrt{x} - 3 \sin 3x)/\sqrt{1 - (\sqrt{x} + \cos 3x)^2}$   
 35.  $2x + 2 \cos(2x + 1), 6t^5 + 6t^2 + 6t^2 \cos(2t^3 + 3)$   
 37.  $\sin^{-1}(x + 1) + \frac{x}{\sqrt{1 - (x + 1)^2}},$   

$$\left[ \sin^{-1}(y - y^3 + 1) + \frac{(y - y^3)}{\sqrt{1 - (y - y^3 + 1)^2}} \right] (1 - 3y^2)$$
  
 39.  $6x^2/(1 + 4x^6), 6b(a + bt)^2/[1 + 4(a + bt)^6]$   
 41.  $(-\cos 3x)/3 + C$   
 43.  $\sin 4x + \cos 4x + C$   
 45.  $-\cos x^3 + x^2 + C$   
 47.  $-\cos(u + 1) + C$   
 49.  $(1/2)\tan^{-1}(y/2) + C$   
 51. 2  
 53.  $\pi/3$   
 55. (a) Differentiate the right side.  
 (b)  $\pi/2$   
 (c)  $x \cos^{-1} x - \sqrt{1 - x^2} + C$   
 (d)  $x \sin^{-1} 3x + \sqrt{1 - 9x^2}/3 + C$   
 57. (a)  $[-1, 1]$   
 (b)  $[5, 9]$   
 (c)  $-1/3$   
 59. Use the inverse function rule.  
 61. 150/121 meters/second.

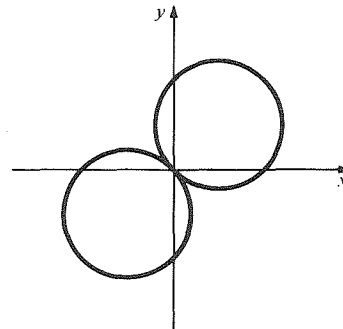
63. (a)  $0, \pi$   
 (b) 0  
 (c) 2.2 meters/second.  
 65. Row to a point  $1/\sqrt{15}$  km downstream.  
 67. 9.798 feet from the sign,  $d\theta/dt = 0$ .  
 69. Inflection points at  $n\pi/2$ ,  $n$  an integer. Concave down on  $[n\pi, (2n + 1)\pi/2]$ ,  $n$  an integer. Concave up elsewhere.



71. Show that  $f'(x)$  is always positive. Deduce that  $x + 1 \geq \cos x$  for  $x \geq 0$ .  
 73.



75.

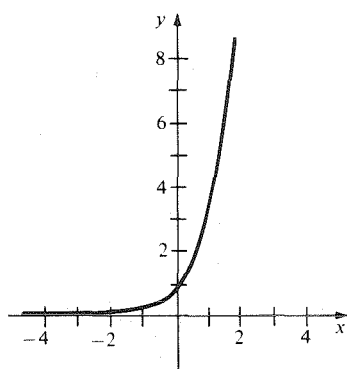


77.  $y = -x - \sqrt{2}$   
 79. (a) On an HP-15c,  $\tan^{-1}[\tan(\pi - 10^{-20})]$  and  $\tan^{-1}[\tan \pi]$  both give  $4.1 \times 10^{-10}$   
 (b)  $-10^{-20}$  is not in the interval.  
 81.  $g'(x) = 1/2f'(f^{-1}(\sqrt{x}))\sqrt{x}$   
 83.  $g''(y) = -f''(x)/[f'(x)]^3$   
 85. To show  $f''(0)$  exists, compute  $[f'(\Delta x) - f'(0)]/\Delta x$  and let  $\Delta x \rightarrow 0$ .

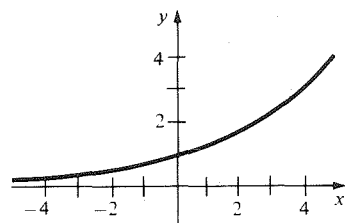
# Chapter 6 Answers

## 6.1 Exponential Functions

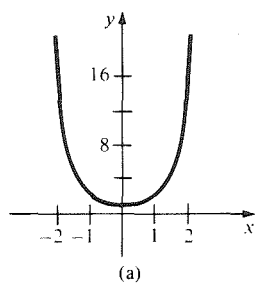
1. 9, 27,  $27\sqrt{3}$ ,  $3^{x/2}$
3. 1.1664, 2.16,  $(1.08)^x$
5. 4
7.  $1 + 3\sqrt{3}$
9.  $5^{5\pi/2} \cdot 6^\pi$
11.  $3^\pi/2^{\sqrt{3}} - 2^{\sqrt{3}}/3^\pi$
13.  $3^{\sqrt{2}}$
15.  $3^{\sqrt{2}}$
- 17.



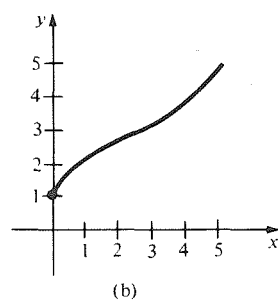
19.



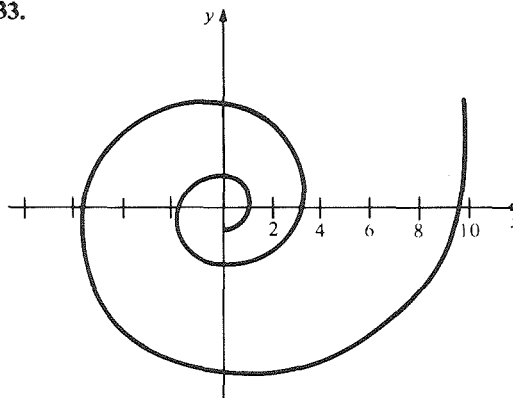
21.



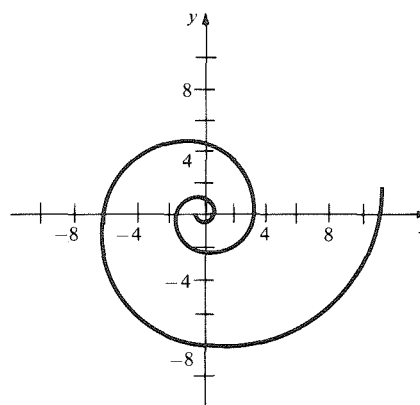
23.



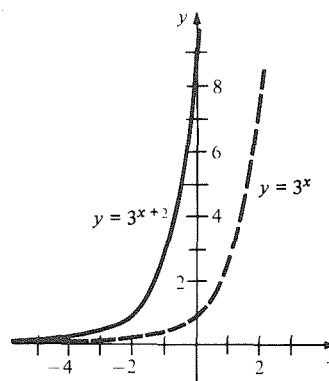
25. Reflect  $y = \exp_3 x$  across the  $y$ -axis to get  $y = \exp_{1/3} x$ .
27. Reflect  $y = \exp_3(-x)$  across the  $y$ -axis to get  $y = \exp_3 x$ .
29. (A)(b), (B)(d), (C)(c), (D)(a)
31. (A)(c), (B)(b), (C)(d), (D)(a)
- 33.



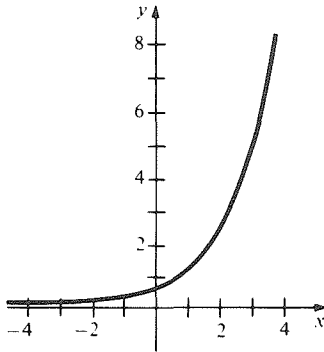
35.



37. The shifting and stretching have the same result since  $3^{x+k} = 3^k 3^x$ .



39.

41.  $1/9$ 43.  $-3$ 

45. No solution

## 6.2 Logarithms

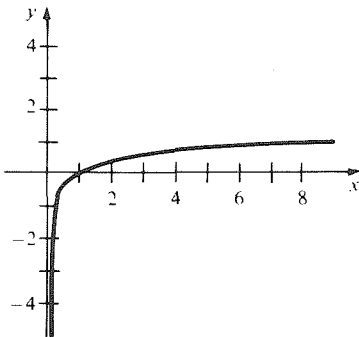
1. 2

3.  $-2$ 5.  $-3$ 

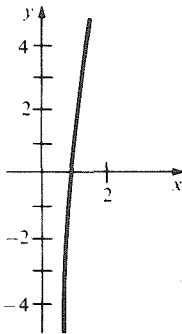
7. 1

9.  $-1$ 

11.



13.



15. (A)(b), (B)(c), (C)(a), (D)(d)

17. 2

19. 4

21.  $b$ 23.  $(2x - 1) - \log_b 2$ 

25. 1.036

27. 0.618

29. 2.51

31.  $b^{y \log_b x} = (b^{\log_b x})^y = x^y$ 33.  $a > 0, \neq 1$ 35.  $2 \log_b A + \log_b B - \log_b C$ 37.  $3 \log_b A - 2 \log_b B + (1/3) \log_b C$ 

39. 3

41. 9

43. 135

45.  $(\sqrt{5} - 1)/2$ 47. Table entries 3.243, 3.486, 3.972; (a)  $\approx 16,410$ ;  
(b)  $b = 1.75$ ,  $M = 1000$ 

49. 0.301, 0.602, 0.903, 1, 2, 3

51. (a) 8.25, 6.7

(b) Use laws of logarithms.

53. 27

55. (a)  $x > -5$ ,  $x \neq \pm 1$ (b)  $(-1, 1)$ (c)  $x > -5$ ,  $x \neq \pm 1$ 

(a) and (c) are the same

57.  $y = 2^x + 1$ ; domain  $(-\infty, \infty)$ , range  $(1, \infty)$ 

## 6.3 Differentiation of the Exponential and Logarithmic Functions

1. 8 times.

3. 0.707 times.

5.  $x + 3$ 7.  $x^3$ 9.  $e^{4x}(4x - 2)$ 11.  $2xe^{x^2+1}$ 13.  $-2xe^{1-x^2} + 3x^2$ 15.  $(\ln 2)2^x + 1$ 17.  $3^x \ln 3 - 2^{x-1} \ln 2$ 19.  $1/x$ 21.  $(1 - \ln x)/x^2$ 23.  $\cot x$ 25.  $2/(2x + 1)$ 27.  $\cos x \ln x + (\sin x)/x$ 29. 
$$\frac{3x(1 + \ln x^2)\sec^2(3x) - 2(\tan 3x)\ln(\tan 3x)}{x \tan 3x(1 + \ln x^2)^2}$$
31.  $1/(\ln 5)x$ 33.  $(\sin x)^x [\ln(\sin x) + x \cot x]$ 35.  $(\sin x)^{\cos x} [\cos^2 x - \sin^2 x \ln(\sin x)]/\sin x$ 37.  $(x - 2)^{2/3}(4x + 3)^{8/7}[2/(3x - 6) + 32/(28x + 21)]$ 39.  $x^{(x^x)} [x^x(1 + \ln x)\ln x + x^{x-1}]$ 41.  $(\sin x + x \cos x)e^{x \sin x}$ 43.  $(x^6 - 5)/(x + x^7)$ 45.  $(2x - 8 \cos x) \ln 14 \cdot 14^{x^2 - 8 \sin x}$ 47.  $(1 + 1/x)/(x + \ln x)$ 49.  $-\sin(x^{\sin x}) \cdot x^{\sin x} [(\cos x)\ln x + (\sin x)/x]$ 51.  $(2x \ln x + x)x^{(x^2)}$ 53.  $-x^{-\tan x^2} \left[ \frac{1}{x} \tan x^2 + 2x(\ln x)\sec^2 x^2 \right]$ 55.  $4x^3 \cos(x^4 + 1) \cdot \log_8(14x - \sin x) + [(14 - \cos x)/(\ln 8)(14x - \sin x)] \sin(x^4 + 1)$ 57.  $3(\ln x/2\sqrt{x} + 1/\sqrt{x})x^{(\sqrt{x})}$ 59.  $\cos x^x \cdot (\ln x + 1)x^x$ 61.  $(\sin x)^{[(\cos x)^x]} (\cos x)^x [(\ln \cos x - x \tan x) \ln \sin x + \cot x]$ 63.  $e^{2x}/2 + C$ 65.  $\sin x + e^{4x}/4 + C$ 67.  $s^3/3 + 2 \ln|s| + C$ 69.  $x^2/4 + (1/2)\ln|x| + C$ 71.  $x + \ln|x - 1| + C$ 73.  $3^x/\ln 3 + C$ 75.  $x^2/2 + 10x + 82 \ln|x - 8| + C$ 77.  $3e - (8/3)$

79.  $65/4 + e^4(e^2 - 1)/2$

81.  $1/\ln 2$

83.  $\ln(3/2)$

85. (a)  $\ln x + 1$ , (b)  $x \ln x - x + C$

87. (a) Differentiate the right side.

(b)  $e^{ax}(b \sin bx + a \cos bx)/(a^2 + b^2) + C$

89. Differentiate the right sides.

91. (a)  $f'(x)e^x + f(x)e^x + g'(x)$

(b)  $(f'(x) + 2x)e^{f(x) + x^2}$

(c)  $f'(x)e^{g(x)} + f(x)g'(x)e^{g(x)}$

(d)  $f'(e^x + g(x))(e^x + g'(x))$

(e)  $f(x)g'(x)(g'(x)\ln f(x) + g(x)f'(x)/f(x))$

93.  $e \approx 2.72$

95. (a) Differentiate using the sum and chain rules.

(b) Use the definitions.

(c) Differentiate  $e^{x+x_0}$ .

97. (a) 0.38629, -0.28345, -0.55765

(b)  $2 \ln 2 - 2 - \lim_{\epsilon \rightarrow 0} \epsilon \ln \epsilon$ ;  $\lim_{\epsilon \rightarrow 0} \epsilon \ln \epsilon = 0$

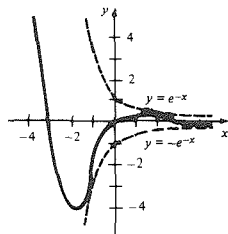
(c)  $\ln x$  has no lower sums on  $[0, 2]$ .

99. Differentiate  $\ln y = \ln f(x) + \ln g(x)$ .

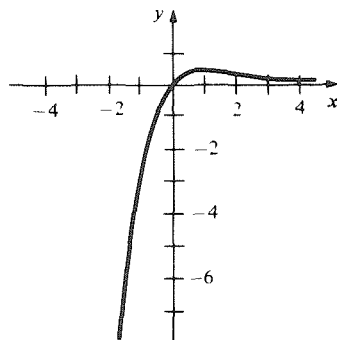
101.  $\frac{dy}{dx} = y \sum_{i=1}^k \left[ \frac{n_i f'_i(x)}{f_i(x)} \right]$

## 6.4 Graphing and Word Problems

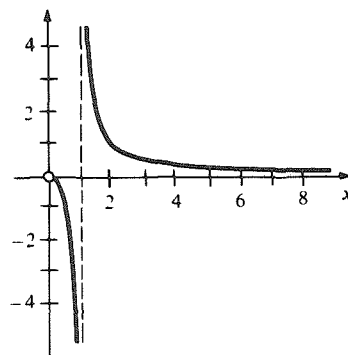
1.



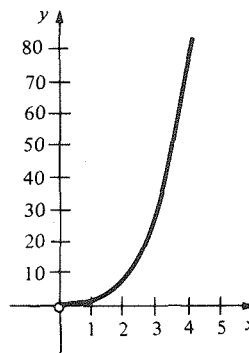
3.



5.



7.



9. 29.6%

11. 0.233%

13.  $\lim_{n \rightarrow \infty} (1 + \sqrt{2} \ln 3/n)^n$

15.  $\lim_{h \rightarrow 0} [3(b^h - 1)/h]$

17. Use formula (8).

19. \$119.72

21.  $y = 3e^{2x} - 2e^2$

23.  $y = -\sqrt{2} \pi x/8 + \sqrt{2}/2$

25.  $y = x + \ln 2 - 1$

27. (a) Compute directly using

$f(x) \approx f(0) + f'(0)(x - 0).$

(b)  $2^{0.01} = 1.00695555$

$1 + 0.01 \ln 2 = 1.00693147$

$2^{0.0001} = 1.000069317$

$1.0001 \ln 2 = 1.000069315$

(c)  $e \approx \left(1 + \frac{1}{n}\right)^n$

29.  $e^{-1/e}$

31. (a) Minimum perceivable sensation.

(b)  $10/\ln 10$

33.  $dp/dt = -0.631$ .

35. Differentiate the expression.

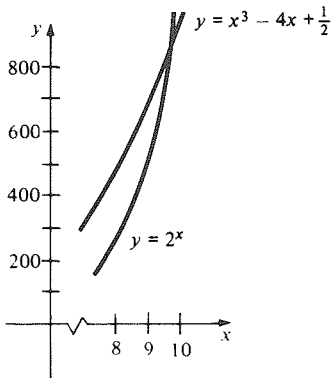
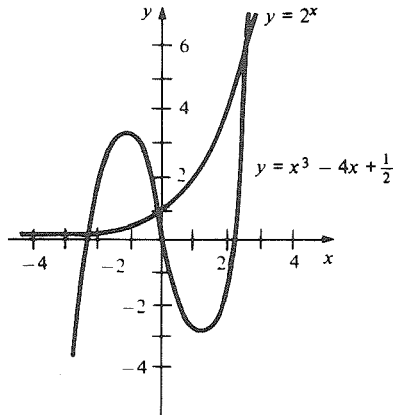
37. (a) Show that  $g''(x) \leq 0$  and that  $\lim_{x \rightarrow \infty} g'(x) = 0$ .

$$37a. (a) \quad g'(x) = f\left(1 + \frac{1}{x}\right) - f'\left(1 + \frac{1}{x}\right) \cdot \frac{1}{x}$$

$$= f\left(1 + \frac{1}{x}\right) - f(1) - f'\left(1 + \frac{1}{x}\right) \cdot \frac{1}{x}$$

Apply the mean value theorem to  $f(1 + 1/x) - f(1)$ .

39. 4



## Review Exercises for Chapter 6

1.  $x^{2\pi} - x^{-2\pi}$
5.  $1/2$
9.  $3x^2e^{x^3}$
11.  $e^x(\cos x - \sin x)$
13.  $e^{\cos 2x}(-2 \sin 2x)$
15.  $2xe^{10x}(1 + 5x)$
17.  $6e^{6x}$
19.  $\frac{e^{\cos x} [(-\sin x)(\cos(\sin x)) + (\cos x)(\sin(\sin x))]}{\cos^2(\sin x)}$
21.  $\frac{[((e^x)\cos(e^x))(e^x + x^2) - (\sin(e^x))(e^x + 2x)]}{(e^x + x^2)^2}$
23.  $(-e^x \sin \sqrt{1 + e^x}) / (2\sqrt{1 + e^x})$
25.  $e^{(\cos x) + x}(-\sin x + 1)$
27.  $-2xe^{-x^2}(x^2 + 2)/(1 + x^2)^2$
29.  $\ln(x + 3) + x/(x + 3)$
31.  $-\tan x$
33.  $1/x \ln 3$
35.  $(-1/\sqrt{1 - (x + e^{-x})^2})(1 - e^{-x})$
37.  $(-2 \ln t)/t[(\ln t)^2 + 3]^2$
39.  $e^{3x}/3 + C$
41.  $\sin x + (1/3)\ln|x| + C$
43.  $x + \ln|x| + C$
45.  $\ln 2 - 2/\pi + 1/2$
47.  $3/2 + \sin 1 - \sin 2 + e - e^2$
49.  $(\ln x)^x(\ln(\ln x) + 1/\ln x)$
51.  $\left[ \frac{(x+3)^{7/2}(x+8)^{5/3}}{(x^2+1)^{6/11}} \right] \left[ \frac{7}{2(x+3)} + \frac{5}{3(x+8)} - \frac{12x}{11(x^2+1)} \right]$
- 53.
- 55.
57.  $(ye^{xy} - 1)/(1 - xe^{xy})$
59.  $-e^{y-x}$
61.  $y = 5x + 1$
63.  $x \sin 6x$ ;  
 $\int x \sin(6x) dx = \sin(6x)/36 - x \cos(6x)/6 + C$
65.  $xe^x/(x+1)^2$ ;  
 $\int [xe^x/(x+1)^2] dx = e^x/(x+1) + C$
67.  $e^8$
69.  $e^{10}$
71. (a) 40% per hour, (b) 40% per hour
73. 7.70%
75. (a) 442 meters.  
(b) In the first 10 seconds, the term contributes 72 meters, in the second, 35 meters.
77. (b) \$11,804.41
79. \$19,876
81. (b)  $\lim_{t \rightarrow 0} P(t) = a/b$
83. Use the mean value theorem on  $(0, x)$ .
85. (a)  $[1 + (b-1)]^n = 1 + n(b-1) + \frac{n(n-1)}{2}(b-1)^2 + \dots \geq 1 + n(b-1)$
87. Use Exercise 86.
89. (a)  $\ln$  is an increasing function.  
(b) Use (a) to deduce  
 $0 \leq \lim_{x \rightarrow \infty} (\ln x/x) \leq \lim_{n \rightarrow \infty} (1/n) = 0.$
91. Let  $z = 1/x$ .
93. Let  $f(x) = x - \left(\frac{2}{\ln 3}\right) \ln x$  and note  $f'(x) > 0$  if  $x \geq \frac{2}{\ln 3} \approx 1.82$ . It is actually valid if  $x \geq 5.8452\dots$